

## UNIT I BASIC CIRCUITS ANALYSIS AND NETWORK TOPOLOGY FUNDAMENTALS OF COMMUNICATION ENGINEERING

### Networks:

Symmetrical and asymmetrical networks. characteristic impedance and propagation constant Derivation of characteristic impedance for T and Pi networks using  $Z_{oc}$  and  $Z_{sc}$ , image and iterative impedances - Derivation of  $Z_{i1}$  and  $Z_{i2}$  for asymmetrical T and L networks using  $Z_{oc}$  and  $Z_{sc}$ , Derivation of iterative impedances for asymmetrical T network. Equaliser: types, applications: constant resistance equalizer. ( No derivations)

### **Symmetrical Networks:**

A network in which all devices can send and receive data at the same rates. Symmetric networks support more bandwidth in one direction as compared to the other, and symmetric DSL offers clients the same bandwidth for both downloads and uploads. A lesser used definition for symmetric network involves resource access—in particular, the equal sharing of resource access.

### **Antenna:**

Basic antenna principle, directive gain, directivity, radiation pattern, broad-side and end -fire array, Yagi antenna, Parabolic antenna.

### **Antenna Directivity:**

Directivity is an important quality of an antenna. It describes how well an antenna concentrates, or bunches, radio waves in a given direction. A dipole transmits or receives most of its energy at right angles to the lengths of metal, while little energy is transferred along them.

If the dipole is mounted vertically, as is common, it will radiate waves away from the center of the antenna in all directions. However, for a commercial radio or television station, a transmitting antenna is often designed to concentrate the radiated energy in certain directions and suppress it in others.

For instance, several dipoles can be used together if placed close to one another. Such an arrangement is called a multiple-element antenna, which is also known as an array.

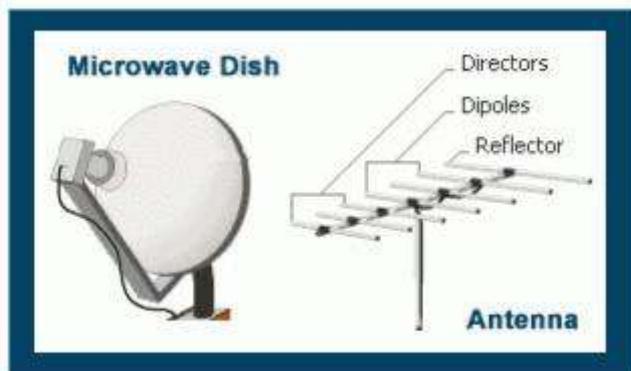
By properly arranging the separate elements and by properly feeding signals to the elements, the broadcast waves can be more efficiently concentrated toward an intended audience, without, for example, wasting broadcast signals over uninhabited areas.

### **Basic Antenna principle:**

#### **Antenna:**

Antenna, also referred to as an aerial, device used to radiate and receive radio waves through the air or through space. Antennas are used to send radio waves to distant sites and to receive radio waves from distant sources. Many wireless communications devices, such as radios, broadcast television sets, radar, and cellular radio telephones, use antennas. Receiving antennas come in many different shapes, depending on the frequency and wavelength of the intended signal.

### **How Antenna works?**



A transmitting antenna takes waves that are generated by electrical signals inside a device such as a radio and converts them to waves that travel in an open space. The waves that are generated by the electrical signals inside radios and other devices are known as guided waves, since they travel through transmission lines such as wires or cables.

The waves that travel in an open space are usually referred to as free-space waves, since they travel through the air or outer space without the need for a transmission line. A receiving antenna takes free-space waves and converts them to guided waves.

Radio waves are a type of electromagnetic radiation, a form of rapidly changing, or oscillating, energy. Radio waves have two related properties known as **frequency** and

**wavelength.**

**Frequency** refers to the number of times per second that a wave oscillates, or varies in strength.

**The wavelength** is equal to the speed of a wave (the speed of light, or 300 million m/sec) divided by the frequency. Low-frequency radio waves have long wavelengths (measured in hundreds of meters), whereas high-frequency radio waves have short wavelengths (measured in centimeters).

An antenna can radiate radio waves into free space from a transmitter, or it can receive radio waves and guide them to a receiver, where they are reconstructed into the original message. For example, in sending an AM radio transmission, the radio first generates a carrier wave of energy at a particular frequency. The carrier wave is modified to carry a message, such as music or a person's voice.

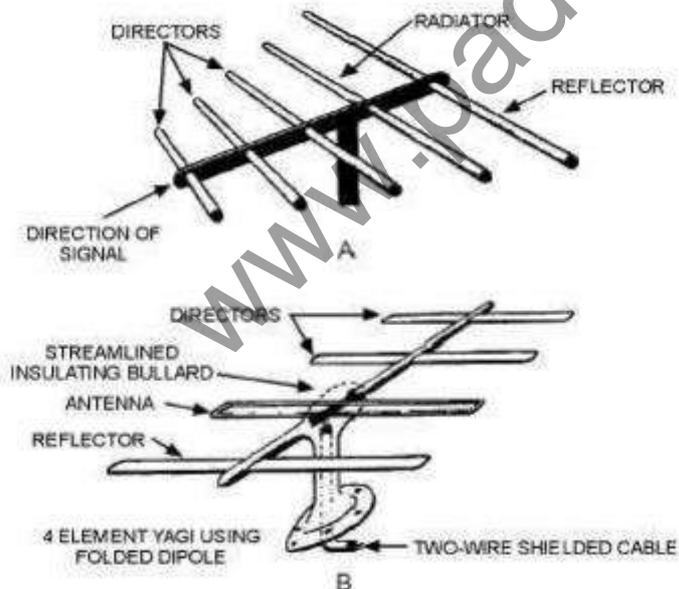
The modified radio waves then travel along a transmission line within the radio, such as a wire or cable, to the antenna. The transmission line is often

known as a feed element. When the waves reach the antenna, they oscillate along the length of the antenna and back. Each oscillation pushes electromagnetic energy from the antenna, emitting the energy through free space as radio waves.

The antenna on a radio receiver behaves in much the same way. As radio waves traveling through free space reach the receiver's antenna, they set up, or induce, a weak electric current within the antenna. The current pushes the oscillating energy of the radio waves along the antenna, which is connected to the radio receiver by a transmission line. The radio receiver amplifies the radio waves and sends them to a loudspeaker, reproducing the original message.

### Yagi antenna:

The Yagi antenna or more correctly, the Yagi - Uda antenna was developed by Japanese scientists in the 1930's. It consists of a half wave dipole (sometimes a folded one, sometimes not), a rear "reflector" and may or may not have one or more forward "directors". These are collectively referred to as the "elements".



The Yagi antenna consists of 2 parts:  
the antenna elements the antenna boom

There are three types of elements:

the Reflector (REFL)

the Driven Element (DE)

the Directors (DIR)

**Yagi antenna components:**

Each Yagi antenna consists of dipoles, reflectors and directors. A dipole antenna receives radio frequency energy in a circular field ending at the center of the dipole. The Yagi antenna uses a series of dipoles in order to allow for a wider range of single to reach the antenna.

With a Yagi antenna all parts of the antenna usually lay on the same plane. This can be extremely useful, especially with more modern Yagi antennas. The more dipoles that the Yagi antenna has on the same plane, the more bands of signal it can pick up at the same time

The Reflector is at the back of the antenna furthest away from the transmitting station. In other words the boom of the antenna is pointed towards the radio station over the horizon with the Reflector furthest away from the station.

The Driven Element is where the signal is intercepted by the receiving equipment and has the cable attached that takes the received signal to the receiver.

**Amplitude Modulation:**

**MODULATION/DEMODULATION:**

Modulation is the process of varying some characteristic of a periodic wave with an external signal. Modulation is the modifying of a signal to carry intelligent data over the communications channel. Several types of modulation are available, depending on the system requirement and equipment. The most frequently used types of modulation are amplitude modulation, frequency modulation, and phase modulation.

Demodulation is the act of returning modulated data signals to their original form.

### 1. Amplitude modulation(AM):

Amplitude modulation refers to modifying the amplitude of a sine wave to store data.

### 2. Frequency Modulation (FM):

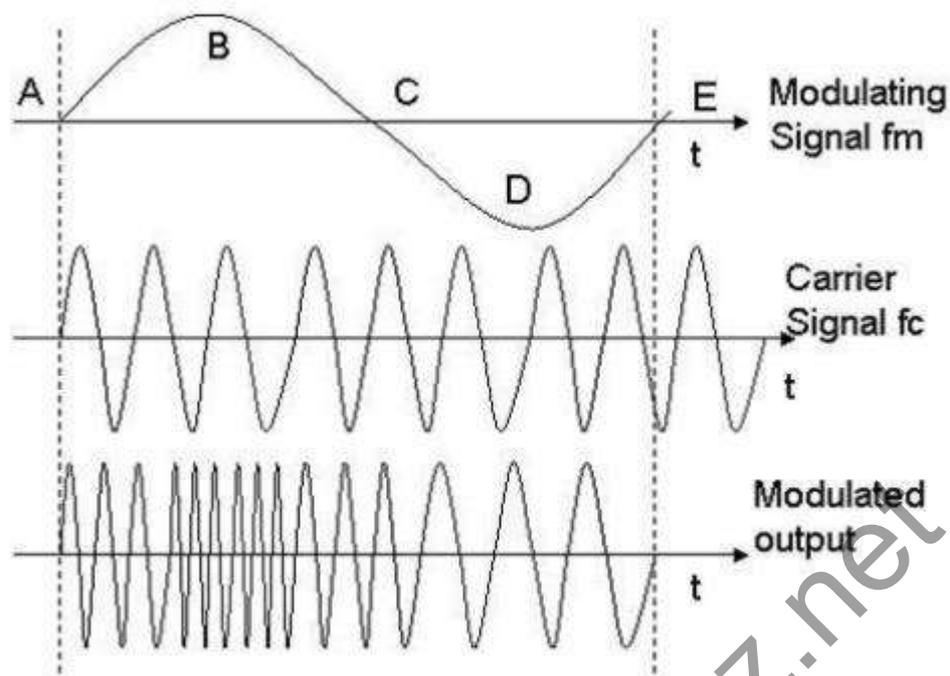
Frequency modulation refers to changing the frequency of a signal to indicate a logic 1 or a logic 0. One frequency indicates a logic 1, and the other frequency indicates a logic 0.

### 3. Phase Modulation (PM or Indirect FM):

Phase modulation is more complex than amplitude modulation or frequency modulation. Phase modulation uses a signal frequency sine wave and performs phase shifts of the sine wave to store data. A modification of phase modulation involves the use of several discrete phase shifts to indicate the state of two or more data bits.

### **Frequency Modulation:**

Frequency Modulation (FM) With frequency modulation, the modulating signal and the carrier are combined in such a way that causes the carrier FREQUENCY( $f_c$ ) to vary above and below its normal(idling) frequency. The amplitude of the carrier remains constant as shown in figure below.



## Microphones:

### Introduction:

A microphone is a transducer as it converts sound waves (acoustic energy) into electrical energy. The very first microphone was purely mechanical in nature. A metal diaphragm is connected to a needle, which —draws a pattern on a metallic foil. When the air pressure changes due to a person's voice, the diaphragm vibrates and moves the needle. The needle then scratches the foil with the vibration pattern. The sound is recreated when the needle is made to run over the foil again. The vibration pattern being followed by the needle makes the diaphragm move and reproduces the sound.

Microphones now work the same way but does the process electronically. Instead of a scratched foil with the vibration patterns, the change in air pressure is now converted to an electrical signal. The diaphragms can be of any material such as plastic, paper or aluminum. Diaphragms differ in producing sound which gave rise to different classifications of microphones

## **Types of microphones:**

### **Carbon microphones:**

Carbon microphones are amongst the oldest, simplest and most used types of microphones even to this day. They work by converting air pressure variations into electrical resistance. The membrane collecting the sound waves presses against a carbon dust material that varies its electrical resistance in the process. By running electric current through the carbon dust, one can obtain an electrical current variation that is amplified and recorded.

### **Condenser Micorphones:**

Condenser microphones rely on the properties of capacitors. However, the plates of the capacitor are no longer immobile and are free to move in relation to each other according to the air pressure changes. This generates a variation in the capacity of the device, which can be converted into electric signals.

### **Dynamic microphones:**

Dynamic microphones on the other hand harness the electromagnetic effects determined by the movement of a magnet inside a conductive wire coil. The vibrations of the magnet are basically converted into tiny electrical currents that are amplified and recorded.

### **Ribbon microphones:**

Ribbon microphones work on a principle rather similar to that of the dynamic microphones, but instead of vibrating a microphone inside a coil, a thin ribbon is suspended in a magnetic field. The vibration of the ribbon translates into inductance variations inside the coil generating the magnetic field.

### **Piezo-electric microphones:**

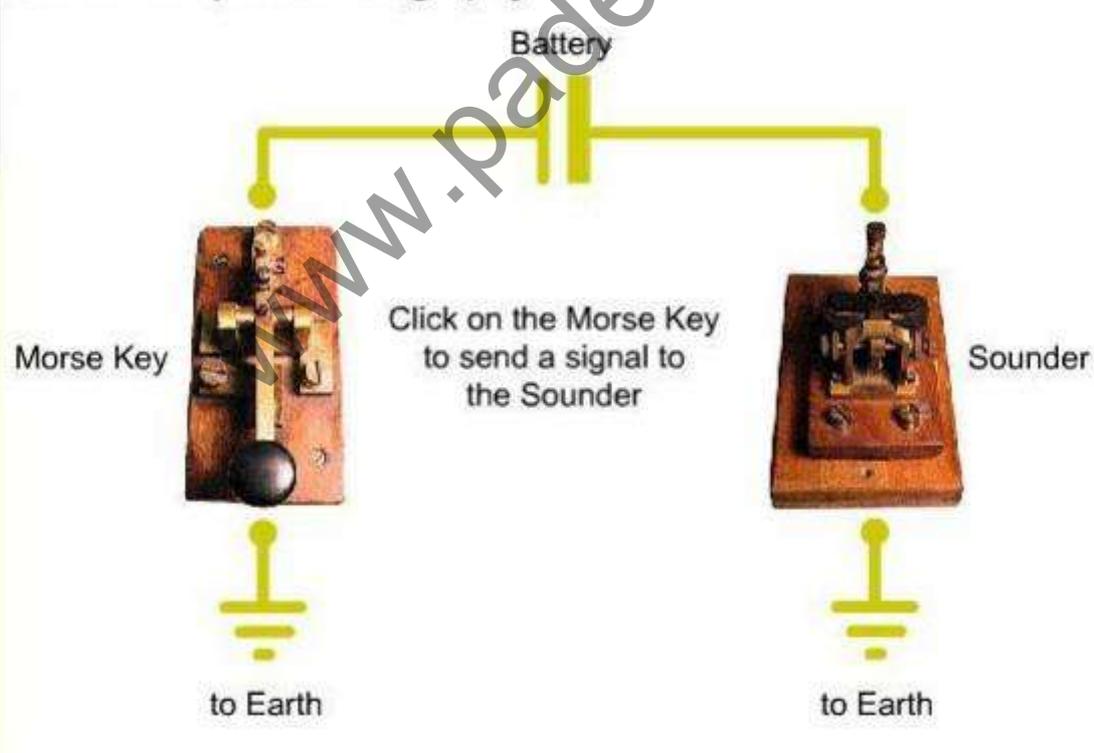
Crystal microphones are based on the piezoelectric effect. Piezoelectric materials have the ability of directly converting electric energy into mechanical movement and vice versa. The most common piezoelectric material occurring naturally on Earth is quartz, which is often used to make crystal microphones.

## Telegraphy

Telegraphy is the long-distance transmission of written messages without physical transport of letters. **Radiotelegraphy** or **wireless telegraphy** transmits messages using radio. Telegraphy includes recent forms of data transmission such as fax, email, and computer networks in general.

A **telegraph** is a machine for transmitting and receiving messages over long distances. A telegraph message sent by a telegraph operator (or telegrapher) using Morse code was known as a **telegram** or **cablegram**, often shortened to a *cable* or a *wire* message. Later, a telegram sent by the Telex network, a switched network of teleprinters similar to the telephone network, was known as a **telex** message.

### Basic Principles of Telegraphy



**Morse Code:**

Morse code is a type of character encoding that transmits telegraphic information using rhythm. Morse code uses a standardized sequence of short and long elements to represent the letters, numerals, punctuation and special characters of a given message. The short and long elements can be formed by sounds, marks, or pulses, in on off keying and are commonly known as "dots" and "dashes" or "dits" and "dahs". The speed of Morse code is measured in words per minute or characters per minute, while fixed-length data forms of telecommunication transmission are usually measured in baud or bps.

### **Television:**

Television (TV) is a widely used telecommunication medium for transmitting and receiving moving images, either monochromatic ("black and white") or color, usually accompanied by sound. "Television" may also refer specifically to a television set, television programming or television transmission.



### **Charge-Coupled Device**

Charge-coupled device (CCD) is an analog shift register that enables the transportation of analog signals (electric charges) through successive stages (capacitors), controlled by a clock signal. Charge-coupled devices can be used as a form of memory or for delaying samples of analog signals. Today, they are most widely used in arrays of photoelectric light sensors to serialize parallel analog signals. Not all image sensors use CCD technology; for example, CMOS chips are also commercially available.

"CCD" refers to the way that the image signal is read out from the chip. Under the control of an external circuit, each capacitor can transfer its electric charge to one or another of its neighbors. CCDs are used in digital photography, digital photogrammetry, astronomy (particularly in photometry), sensors, electron

microscopy, medical fluoroscopy, optical and UV spectroscopy, and high speed techniques such as lucky imaging.

Television is certainly one of the most influential forces of our time. Through the device called a television set or TV, you are able to receive news, sports, entertainment, information and commercials. The average American spends between two and five hours a day glued to "the tube"!

Have you ever wondered about the technology that makes television possible? How is it that dozens or hundreds of channels of full-motion video arrive at your house, in many cases for free? How does your television decode the signals to produce the picture? How will the new digital television signals change things? If you have ever wondered about your television (or, for that matter, about your computer monitor), then read on! In this article, we'll answer all of these questions and more. See the next page to get started.

conversion of the vibrations of sound (for example, music) into a permanent record, and its later playback in its original form (see SOUND,). In the most common method of sound recording, the magnetic method, transformed sound waves may be amplified and made to magnetize a metaloxide coated plastic recording tape so that the magnetization varies with the frequency and intensity of the sound. Sound recording involves some form of mechanical movement of the recording medium at a constant speed past the point of recording so that the sound recording may later be reproduced as a replica of the original sound.

## **Components of Television**

### **HIGH FIDELITY**

High fidelity is the technique of recording, broadcasting, and reproducing sound to match as closely as possible the characteristics of the original sound. To achieve high-fidelity reproduction, the sound must be free of distortion and include the full frequency range of human hearing—20 Hz to 20 kilohertz (see FREQUENCY,).

#### **Components.**

A high-fidelity system consists of the following components: the turntable and tonearm or possibly a CD player, the amplifier, the speaker system, and the control unit, sometimes referred to as a preamplifier/control unit. Supplementary components include the tuner and the tape recorder.

### **The turntable and tonearm.**

(For the basic operating principles of these elements, *see* PHONOGRAPH.) The turntable and tonearm translate the engraved patterns on a phonograph record into electrical voltage variations. The turntable is rotated by a motor that turns at a constant speed, thus avoiding distortions called wow and rumble. Wow consists of a slow variation in pitch caused by variation in the speed of the turntable, and rumble is a low-frequency tremor caused by defects in the turntable.

The tonearm and the cartridge form one of the most critical parts of the high-fidelity installation. The finely balanced tonearm holds a cartridge, which in turn holds a stylus, preferably tipped with long-wearing diamond. To reproduce recorded sound accurately and with minimum wear on the record, the cartridge must provide maximum compliance, that is, an easy lateral and vertical motion of the stylus. The stylus, moreover, must contact the record at a precise angle with the proper pressure.

### **The compact disc (CD) player.**

CD players are increasingly replacing the conventional turntable and tonearm in high-fidelity systems. Offering more uniform frequency response, lower distortion, and inaudible background noise levels, compact discs have the additional advantage of longer life. Since CDs are never physically in contact with any pickup mechanism—digital codes embedded beneath the surface of the disc are read by a laser beam of light—these discs can last indefinitely if handled with care. Specially built CD players can also be used for data retrieval

using CD-ROM (Read-Only Memory) discs, while interactive compact discs (CD-I), as well as interactive video discs (VD-I), can be used for a wide variety of educational and training purposes. In addition to their audio content, some CDs contain digitally driven graphics that can be displayed on a television screen. Such discs are referred to as CD-G.

### **The amplifier.**

The amplifier converts the relatively weak electrical impulses received from the cartridge into power sufficient to drive the speakers. The amount of power that an amplifier can produce is rated in watts. Depending on the requirements of the speaker system, an amplifier may deliver from 10 to 125 watts of electrical power. The amplifier is controlled, as a rule, by a device called the preamplifier, which amplifies minute sound-signal voltages too small for the amplifier to handle. Preamplifiers also boost the bass and attenuate the treble to compensate for the poor bass and strong treble response of phonograph records. Most modern amplifiers are equipped with so-called solid-state or integrated circuits. *See* INTEGRATED CIRCUIT.

### **The speaker system.**

Loudspeakers, electromechanical devices that produce audible sound from amplified audio voltages, are extensively employed in radio receivers, motion picture sound systems, public-address systems, and other apparatus in which sound must be produced from a recording, a communications system, or a sound source of low intensity.

Several types of loudspeaker exist, but almost all loudspeakers now in use are dynamic speakers. These speakers include an extremely light coil of wire, called the voice coil, mounted within the magnetic field of a powerful permanent magnet or electromagnet. The coil of the electromagnet, if one is used, is called the field

coil. A varying electric current from the amplifier passes through the voice coil and alters the magnetic force between the voice coil and the speaker's magnetic field. As a result, the coil vibrates with the changes in the current. A diaphragm or a large paper cone mechanically attached to the voice coil generates sound waves in the air when the coil moves.

The loudness and sound quality of such speakers can be increased by the use of properly designed enclosures or cabinets. Such cabinets may hold several loudspeakers of different sizes, small so-called tweeters for high notes, and large woofers for low notes.

### **The control unit.**

As the nerve center of the high-fidelity system, the control unit performs a number of critical functions. For example, the surface noises of old records are attenuated by means of a device called the scratch filter; another device, the rumble filter, cuts down low-pitched noises, such as vibration from the phonograph motor; the loudness control compensates for the inability of the ear to hear high and low notes as clearly as it hears the middle range by increasing the relative level of treble and bass tones when the record is played at a reduced volume. The control unit also adjusts sound signals from the record player, the tape recorder, or the tuner.

### **The tuner.**

The AM/FM tuner allows the listener to receive broadcasts from stations in the broadest band of the radio spectrum, from 500 to 1650 kilohertz (AM), 88 to 108 megahertz (FM). From the broadcast signals reaching the antenna, the tuner selects the frequency of the desired station to the exclusion of other stations in

the broadcast range. It then extracts the audio voltage representing the program being transmitted and amplifies this voltage to activate the speakers of the high-fidelity system. *See* RADIO,.

### **The tape recorder.**

This device records and reproduces sound by preserving electrical signals as magnetic patterns on thin plastic tape coated with magnetic oxide. In recording, the tape is drawn past a recording head, leaving a magnetic imprint. The tape is then drawn past a reproducing head that turns the magnetic pattern into an electrical signal; this signal, in turn, is amplified and reproduced as sound. The reproducing, or playback, head may be the same device as the recording head, or they may be separate devices. Tapes are readily erased for reuse and are virtually immune to the damage that eventually mars phonograph records.

The first magnetic-reading instrument, called a telegraphone, was invented in 1898 by the Danish electrical engineer Valdemar Poulsen (1869–1942), who used a magnetized steel tape to carry messages. Currently, the most popular form of tape recording is the so-called compact cassette, which uses a tape with two or four tracks. Cassette-tape recorders and players are available in a wide variety of sizes, from the tiny portable types used with stereo headphones to elaborate units incorporated in home high-fidelity systems.

### **STEREOPHONIC SOUND**

Stereophonic sound re-creates for listeners the conditions that would exist near an actual sound source, such as an orchestra. The sound is picked up separately from the left and the right sides of the orchestra, and, through the use of two or more carefully placed speakers, a stereophonic recording is directed toward listeners in such a way that they seem to hear music from the left, the right, and the center. More importantly, they become aware of a veil of sound that seems to have depth and solidity as well as direction.

## TYPES OF RECORDING:

### **Mechanical Recording.**

The operation of a sound-recording system may, however, be most easily understood by considering the process of recording sound by the now obsolete mechanical method. In this method, sound waves are used directly or indirectly to actuate a stylus or cutter that engraves on a disk or cylinder a wavy-line pattern corresponding to the pattern of sound waves. This process, with minor modifications, was used for many years in the production of phonograph records. In the direct method of mechanical recording, sound waves strike a very light diaphragm of metal or other substance and set it into motion. Attached to the diaphragm is a needle or cutting point that vibrates with the diaphragm. Under the point is a disk or cylinder of wax, metallic foil, shellac, or other suitable substance that is moved past the needle so that the needle cuts a groove in the form of a spiral on a disk, or a helix on a cylinder. As the needle vibrates it traces a wavy groove laterally or vertically in the record; this groove is a mechanical replica of the sound that struck the diaphragm of the recording machine. If, for example, the sound wave consists of the musical tone of A in the treble clef, which has a frequency of 440 Hz (hertz or cycles per second), the needle oscillates 440 times/sec. If the record is moving under the needle at the rate of 10 cm/sec, the groove will exhibit a pattern of 44 oscillations (44 sine waves, or 44 crests and 44 troughs)/1 cm (0.4 in). To reproduce the recorded sound, a needle attached to a diaphragm is set in the groove, and the record is turned at the rate of 10 cm (4 in)/sec. The vertical or, more commonly, lateral crests and troughs of groove then move the needle at the rate of 440 oscillations/sec, and the attached

diaphragm oscillates, producing sound waves in the air of the same pitch as the original tone (*see* OSCILLATION,). In the making of modern phonograph records the sound is first converted to electrical impulses by a microphone; these impulses are amplified and used to actuate the cutting needle by electromagnetic means. The cutting needle engraves a disk, called the master, made of shellac, which is used to make the metal molds from which vinyl records are mass produced.

### **Optical Recording.**

In the optical method, sound waves are transformed by a microphone into equivalent electrical impulses that are then amplified and made to operate a device that changes the intensity of a light beam (by means of an electromagnetically actuated gate or light valve) or the size of the beam (by means of an electromagnetically actuated vibrating mirror or a slit of variable width). The resulting varying light beam is focused on a moving film, which is then developed to provide a photographic track. The track recorded with varying intensity is of variable density and constant width. The track recorded with the vibrating mirror or varying slit has variable areas of darkened and clear film. To reproduce the sound track on either film, a light source is focused on the film, and a PHOTOELECTRIC CELL, (q.v.) is placed behind the film. The fluctuations in the relative amount of light passing through the film generate a fluctuating electric current in the photoelectric cell; this current is amplified and then transformed into sound by means of some form of loudspeaker. *See* MOTION PICTURE,.

### **Electromagnetic Recording.**

In audiotape recording, sound waves are amplified and recorded on a magnetized plastic or paper tape. The information is first converted into electrical impulses, which are then impressed in the magnetized tape by an electromagnetic record head. A playback head, which is also an electromagnetic device, converts the magnetic fields on the tape into electrical impulses that are then amplified and reconverted into audible sound waves.

### **Digital Recording.**

In the combined mechanical and electronic system of ordinary phonograph recording, waveforms of sound are inevitably distorted to some degree, and they also pick up noises from the recording process itself. In computer-based recording these problems are eliminated. The digital recorder measures the waveforms thousands of times each second and assigns a numerical value, or digit, to each of the measurements. These digits are then translated into a stream of electronic pulses that are placed in a memory bank for later retranslation and playback. Such techniques have been used limitedly in recent years for the production of otherwise conventional phonograph records, but direct-digital records are now available in which electronic pulses are instead placed on a small, aluminized disc called a COMPACT DISC (q.v.; CD), where they somewhat resemble a spiral of Morse-code signals when viewed through a microscope. The plastic-protected

CD is placed in a machine where a laser beam reads the coded information, and circuitry converts it to analog signals for playback through conventional speaker systems.

### **Stereophonic Recording.**

Stereophonic recording in its simplest form uses two separate microphones to produce two recorded tracks, or channels, on magnetic tape. Similarly, the sound component of motion pictures reproduces stereophonic sound by multiple tracks on film.

Phonograph disks can also record stereophonic sound, or stereo, on two independent channels, one on each wall of a single groove. The groove is cut with a  $90^\circ$  stylus in such a way that one groove wall slants  $45^\circ$  to the left, and the other wall slants  $45^\circ$  to the right. Two independent coils  $90^\circ$  apart energize the cutting stylus to give a different pattern on each wall for each of the two channels. During playback of a disk, two sensors in the cartridge are mounted at a  $90^\circ$  angle to each other to pick up the two tracks.

### **Quadraphonic Recording.**

A quadraphonic sound playback system requires the use of four separate amplification channels driving four speakers located in the four corners of the listening room. Various systems achieving quadraphonic recording and playback were perfected in the early 1970s, some involving a method of encoding and decoding which required only two channels to be recorded on tape or disk.

The lack of standardization of these systems and the reluctance of many music lovers to place four loudspeakers in the listening room caused the popularity of quadraphonic recording to wane. With the advent of home video recorders, or VCRs, and large-screen television sets in the 1980s, a new type of multi-channel sound has replaced quadraphonic sound. Called surround sound, this system also involves the use of four or more loudspeakers and channels and is used to re-create the all-enveloping sound experienced when attending certain motion pictures in specially equipped theaters.

## **BASIC CIRCUITS ANALYSIS**

1. INTRODUCTION
2. BASIC ELEMENTS & INTRODUCTORY CONCEPTS
3. KIRCHOFF'S LAW
4. PROBLEMS AND CALCULATIONS
5. DC CIRCUITS
6. AC CIRCUITS
7. DIFFERENCE BETWEEN AC AND DC
8. PARALLEL NETWORKS
9. MESH ANALYSIS
10. NODAL ANALYSIS

## **1.INTRODUCTION:**

The interconnection of various electric elements in a prescribed manner comprises as an electric circuit in order to perform a desired function. The electric elements include controlled and uncontrolled source of energy, resistors, capacitors, inductors, etc. Analysis of electric circuits refers to computations required to determine the unknown quantities such as voltage, current and power associated with one or more elements in the circuit. To contribute to the solution of engineering problems one must acquire the basic knowledge of electric circuit analysis and laws. Many other systems, like mechanical, hydraulic, thermal, magnetic and power system are easy to analyze and model by a circuit. To learn how to analyze the models of these systems, first one needs to learn the techniques of circuit analysis. We shall discuss briefly some of the basic circuit elements and the laws that will help us to develop the background of subject.

## **2. BASIC ELEMENTS & INTRODUCTORY CONCEPTS:**

### **Electrical Network:**

A combination of various electric elements (Resistor, Inductor, Capacitor, Voltage source, Current source) connected in any manner what so ever is called an electrical network. We may classify circuit elements in two categories, passive and active elements.

### **Passive Element:**

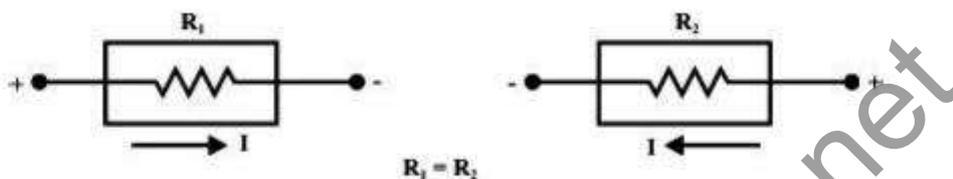
The element which receives energy (or absorbs energy) and then either converts it into heat (R) or stored it in an electric (C) or magnetic (L) field is called passive element.

### **Active Element:**

The elements that supply energy to the circuit is called active element. Examples of active elements include voltage and current sources, generators, and electronic devices that require power supplies. A transistor is an active circuit element, meaning that it can amplify power of a signal. On the other hand, transformer is not an active element because it does not amplify the power level and power remains same both in primary and secondary sides. Transformer is an example of passive element.

### Bilateral Element:

Conduction of current in both directions in an element (example: Resistance; Inductance; Capacitance) with same magnitude is termed as bilateral element.



### Unilateral Element:

Conduction of current in one direction is termed as unilateral (example: Diode, Transistor) element.



### Meaning of Response:

An application of input signal to the system will produce an output signal, the behavior of output signal with time is known as the response of the system.

### Potential Energy Difference:

The voltage or potential energy difference between two points in an electric circuit is the amount of energy required to move a unit charge between the two points.

### Ohm's Law:

Ohm's law states that the current through a conductor between two points is directly proportional to the potential difference or voltage across the two points, and inversely proportional to the resistance between them. The mathematical equation that describes this relationship is:

$$I = V / R$$

$$I = \frac{V}{R}$$

where I is the current through the resistance in units of amperes, V is the potential difference measured across the resistance in units of volts, and R is the resistance of the conductor in units of ohms. More specifically, Ohm's law states that the R in this relation is constant, independent of the current.

### 3. KIRCHOFF'S LAW

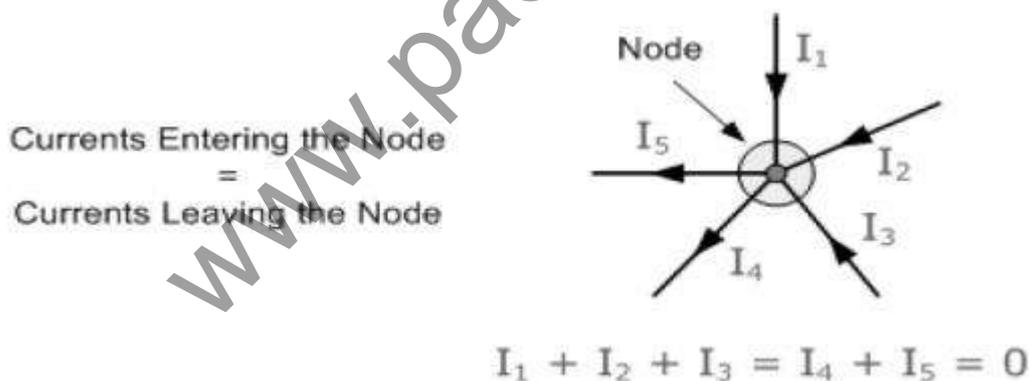
#### Kirchoff's First Law - The Current Law, (KCL)

"The total current or charge entering a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node".

In other words the algebraic sum of ALL the currents entering and leaving a node must be equal to zero,

$$I(\text{exiting}) + I(\text{entering}) = 0.$$

This idea by Kirchoff is known as the Conservation of Charge.



Here, the 3 currents entering the node, I1, I2, I3 are all positive in value and the 2 currents leaving the node, I4 and I5 are negative in value.

Then this means we can also rewrite the equation as;  $I_1 + I_2 + I_3 - I_4 - I_5 = 0$

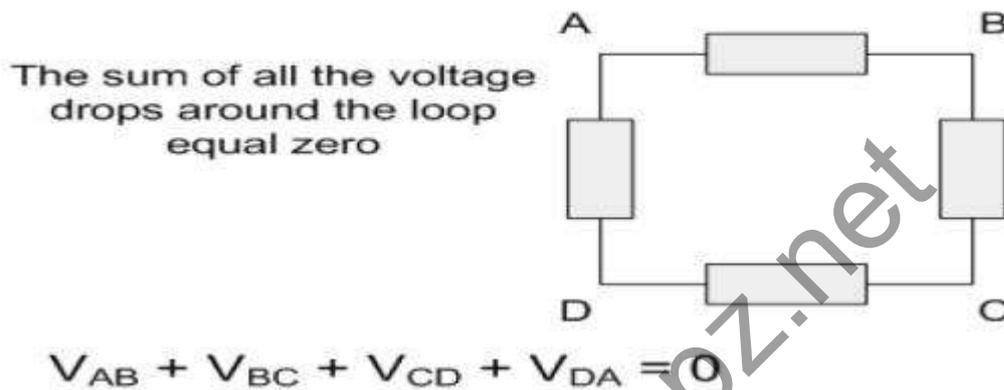
#### Kirchoff's Second Law - The Voltage Law, (KVL)

"In any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop" which is also equal to zero.

In other words the algebraic sum of all voltages within the loop must be equal to zero. This idea by Kirchoff is known as the Conservation of Energy.

Starting at any point in the loop continue in the same direction noting the direction of all the voltage drops, either positive or negative, and returning back to the same starting point. It is important to maintain the same direction either clockwise or anti-clockwise or the final voltage sum will not be equal to zero.

We can use Kirchoff's voltage law when analyzing series circuits.



**Problem 1:**

A current of 0.5 A is flowing through the resistance of  $10\Omega$ . Find the potential difference between its ends.

Solution:

Current  $I = 0.5\text{A}$ .

Resistance  $R = 10\Omega$

Potential difference  $V = ?$

$$\begin{aligned} V &= IR \\ &= 0.5 \times 10 \\ &= 5\text{V}. \end{aligned}$$

**Problem: 2**

A supply voltage of 220V is applied to a resistor  $100\Omega$ . Find the current flowing through it.

**Solution:**

$$\text{Voltage } V = 220\text{V Resistance } R = 100\Omega \text{ Current } I = V / R$$

$$= 220 / 100$$

$$= 2.2 \text{ A.}$$

**Problem: 3**

Calculate the resistance of the conductor if a current of 2A flows through it when the potential difference across its ends is 6V.

**Solution:**

$$\text{Current } I = 2\text{A. Potential difference } = V = 6. \text{ Resistance } R = V/I$$

$$= 6 / 2$$

$$= 3 \text{ ohm.}$$

**Problem: 4**

Calculate the current and resistance of a 100 W, 200V electric bulb.

**Solution:**

$$\text{Power, } P = 100\text{W}$$

$$\text{Voltage, } V = 200\text{V Power } p = VI$$

$$\text{Current } I = P/V$$

$$= 100/200$$

$$= 0.5\text{A}$$

$$\text{Resistance } R = V / I$$

$$= 200/0.5$$

$$= 400\text{W.}$$

**Problem: 5**

Calculate the power rating of the heater coil when used on 220V supply *taking 5 Amps.*

**Solution:**

$$\text{Voltage, } V = 220\text{V Current, } I = 5\text{A, Power, } P = VI$$

$$\begin{aligned}
 &= 220 \times 5 \\
 &= 1100\text{W} \\
 &= 1.1 \text{ KW.}
 \end{aligned}$$

**Problem: 6**

A circuit is made of 0.4 wire,  $\Omega$  a 150 bulb  $\Omega$  and a rheostat 120 connected  $\Omega$  in series. Determine the total resistance of the resistance of the circuit.

**Solution:**

Resistance of the wire = 0.4 Resistance  $\Omega$  of bulb = 150  $\Omega$  Resistance of rheostat = 120  $\Omega$

In series,

$$\text{Total resistance, } R = 0.4 + 150 + 120 = 270.4 \Omega$$

**Problem : 7**

Three resistances of values 2  $\Omega$ , 3  $\Omega$  connected and in series 5  $\Omega$  across are 20 V ,D.C supply

.Calculate (a) equivalent resistance of the circuit (b) the total current of the circuit (c) the voltage drop across each resistor and (d) the power dissipated in each resistor.

Solution:

$$\begin{aligned}
 \text{Total resistance } R &= R_1 + R_2 + R_3. \\
 &= 2 + 3 + 5 = 10 \Omega
 \end{aligned}$$

$$\text{Voltage } = 20\text{V}$$

$$\text{Total current } I = V/R = 20/10 = 2\text{A.}$$

$$\begin{aligned}
 \text{Voltage drop across } 2\Omega \text{ resistor } V_1 &= I R_1 \\
 &= 2 \times 2 = 4 \text{ volts.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Voltage drop across } 3\Omega \text{ resistor } V_2 &= I R_2 \\
 &= 2 \times 3 = 6 \text{ volts.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Voltage drop across } 5\Omega \text{ resistor } V_3 &= I R_3 \\
 &= 2 \times 5 = 10 \text{ volts.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Power dissipated in } 2\Omega \text{ resistor is } P_1 &= I^2 R_1 \\
 &= 2^2 \times 2 = 8 \text{ watts.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Power dissipated in } 3 \text{ resistor is } P_2 &= I^2 R_2. \\
 &= 2^2 \times 3 = 12 \text{ watts.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Power dissipated in } 5 \text{ resistor is } P_3 &= I^2 R_3 \\
 &= 2^2 \times 5 = 20 \text{ watts.}
 \end{aligned}$$

**Problem: 8**

A lamp can work on a 50 volt mains taking 2 amps. What value of the resistance must be connected in series with it so that it can be operated from 200 volt mains giving the same power.

**Solution:**

Lamp voltage,  $V = 50\text{V}$  Current,  $I = 2$  amps.

Resistance of the lamp  $= V/I = 50/2 = 25 \Omega$

Resistance connected in series with lamp  $= r$ .

Supply voltage  $= 200$  volt.

Circuit current  $I = 2\text{A}$

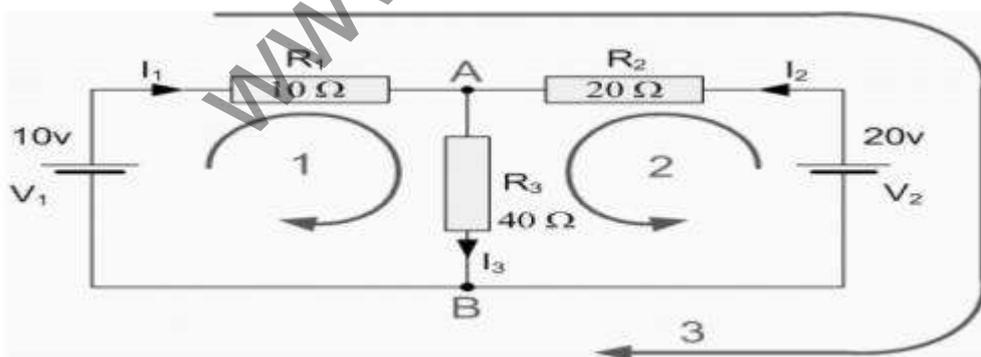
Total resistance  $R_t = V/I = 200/2 = 100\Omega$

$$R_t = R + r \quad 100 = 25 + r$$

$$r = 75\Omega$$

**Problem: 9**

Find the current flowing in the  $40\Omega$  Resistor,



**Solution:**

The circuit has 3 branches, 2 nodes (A and B) and 2 independent loops.

Using Kirchoff's Current Law, KCL the equations are given as;

At node A:  $I_1 + I_2 = I_3$

At node B:  $I_3 = I_1 + I_2$

Using Kirchoff's Voltage Law, KVL the equations are given as;

Loop 1 is given as:  $10 = R_1 \times I_1 + R_3 \times I_3 = 10I_1 + 40I_3$

Loop 2 is given as:  $20 = R_2 \times I_2 + R_3 \times I_3 = 20I_2 + 40I_3$

Loop 3 is given as:  $10 - 20 = 10I_1 - 20I_2$

As  $I_3$  is the sum of  $I_1 + I_2$  we can rewrite the equations as;

Eq. No 1:  $10 = 10I_1 + 40(I_1 + I_2) = 50I_1 + 40I_2$

Eq.No 2:  $20 = 20I_1 + 40(I_1 + I_2) = 40I_1 + 60I_2$

We now have two "Simultaneous Equations" that can be reduced to give us the value of both  $I_1$  and

$I_2$

Substitution of  $I_1$  in terms of  $I_2$  gives us the value of  $I_1$  as  $-0.143$  Amps

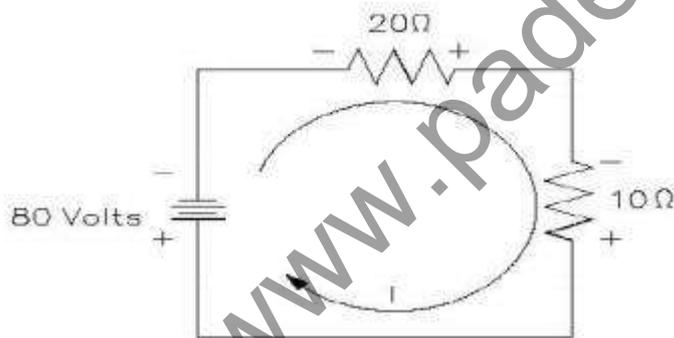
Substitution of  $I_2$  in terms of  $I_1$  gives us the value of  $I_2$  as  $+0.429$  Amps

As:  $I_3 = I_1 + I_2$

The current flowing in resistor  $R_3$  is given as:  $-0.143 + 0.429 = 0.286$  Amps and the voltage across the resistor  $R_3$  is given as :  $0.286 \times 40 = 11.44$  volts

### Problem: 10

Find the current in a circuit using Kirchoff's voltage law



**Solution:**

$$80 = 20(I) + 10(I)$$

$$80 = 30(I)$$

$$I = 80/30 = 2.66 \text{ amperes}$$

## 5. DC CIRCUITS:

A DC circuit (Direct Current circuit) is an electrical circuit that consists of any combination of constant voltage sources, constant current sources, and resistors. In this case, the circuit voltages and currents are constant, i.e., independent of time. More technically, a DC circuit has no memory. That is, a particular circuit

voltage or current does not depend on the past value of any circuit voltage or current. This implies that the system of equations that represent a DC circuit do not involve integrals or derivatives.

If a capacitor and/or inductor is added to a DC circuit, the resulting circuit is not, strictly speaking, a DC circuit. However, most such circuits have a DC solution. This solution gives the circuit voltages and currents when the circuit is in DC steady state. More technically, such a circuit is represented by a system of differential equations. The solutions to these equations usually contain a time varying or transient part as well as constant or steady state part. It is this steady state part that is the DC solution. There are some circuits that do not have a DC solution. Two simple examples are a constant current source connected to a capacitor and a constant voltage source connected to an inductor.

In electronics, it is common to refer to a circuit that is powered by a DC voltage source such as a battery or the output of a DC power supply as a DC circuit even though what is meant is that the circuit is DC powered.

## **6. AC CIRCUITS:**

### **Fundamentals of AC:**

An alternating current (AC) is an electrical current, where the magnitude of the current varies in a cyclical form, as opposed to direct current, where the polarity of the current stays constant.

The usual waveform of an AC circuit is generally that of a sine wave, as these results in the most efficient transmission of energy. However in certain applications different waveforms are used, such as triangular or square waves.

Used generically, AC refers to the form in which electricity is delivered to businesses and residences. However, audio and radio signals carried on electrical wire are also examples of alternating current. In these applications, an important goal is often the recovery of information encoded (or modulated) onto the AC signal.

## **7. DIFFERENCE BETWEEN AC AND DC:**

Current that flows continuously in one direction is called direct current. Alternating current (A.C) is the current that flows in one direction for a brief time then reverses and flows in opposite direction for a similar time. The source for alternating current is called a.c generator or alternator.

### **Cycle:**

One complete set of positive and negative values of an alternating quantity is called cycle.

**Frequency:**

The number of cycles made by an alternating quantity per second is called frequency. The unit of frequency is Hertz (Hz)

**Amplitude or Peak value:**

The maximum positive or negative value of an alternating quantity is called amplitude or peak value.

**Average value:**

This is the average of instantaneous values of an alternating quantity over one complete cycle of the wave.

**Time period:**

The time taken to complete one complete cycle.

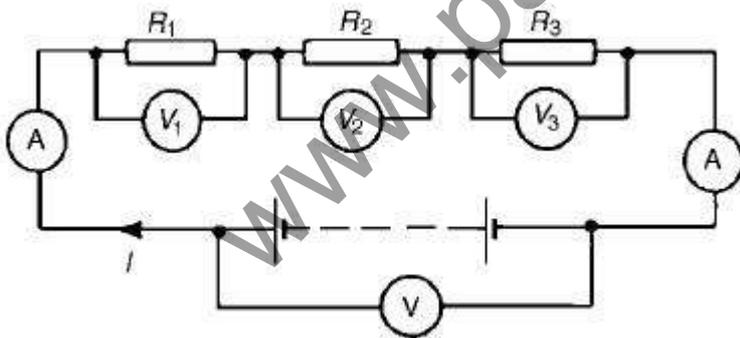
**Average value derivation:**

Let  $i$  = the instantaneous value of current and  $i = I_m \sin \theta$  Where,  $I_m$  is the maximum value.

**Resistors in series and parallel circuits:**

**Series circuits:**

Figure shows three resistors  $R_1$ ,  $R_2$  and  $R_3$  connected end to end, i.e. in series, with a battery source of  $V$  volts. Since the circuit is closed a current  $I$  will flow and the p.d. across each resistor may be determined from the voltmeter readings  $V_1$ ,  $V_2$  and  $V_3$



**In a series circuit**

(a) the current  $I$  is the same in all parts of the circuit and hence the same reading is found on each of the two ammeters shown, and

(b) the sum of the voltages  $V_1$ ,  $V_2$  and  $V_3$  is equal to the total applied voltage,  $V$ , i.e.

$$V = V_1 + V_2 + V_3$$

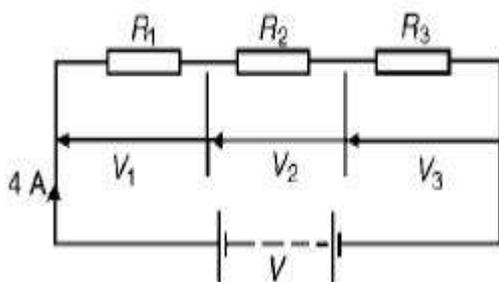
From Ohm's law:

$V_1 = IR_1$ ,  $V_2 = IR_2$ ,  $V_3 = IR_3$  and  $V = IR$  where  $R$  is the total circuit resistance.  
 Since  $V = V_1 + V_2 + V_3$

then  $IR = IR_1 + IR_2 + IR_3$  Dividing throughout by  $I$  gives  $R = R_1 + R_2 + R_3$

Thus for a series circuit, the total resistance is obtained by adding together the values of the separate resistances.

**Problem 1:** For the circuit shown in Figure 5.2, determine (a) the battery voltage  $V$ , (b) the total resistance of the circuit, and (c) the values of resistance of resistors  $R_1$ ,  $R_2$  and  $R_3$ , given that the p.d.'s  $R_1$ ,  $R_2$  and  $R_3$  are  $5V$ ,  $2V$  and  $6V$  respectively.



(a) Battery voltage  $V = V_1 + V_2 + V_3 = 5 + 2 + 6 = 13V$

(b) Total circuit resistance  $R = V / I$   
 $= 13 / 4 = 3.25 \Omega$

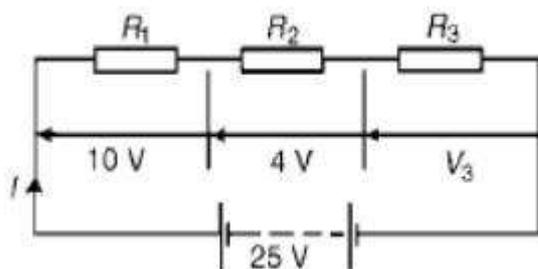
(c) Resistance  $R_1 = V_1 / I$   
 $= 5 / 4$

$= 1.25 \Omega$  Resistance  $R_2 = V_2 / I$

$= 2 / 4 = 0.5 \Omega$

Resistance  $R_3 = V_3 / I = 6 / 4 = 1.5 \Omega$

**Problem 2.** For the circuit shown in Figure determine the p.d. across resistor  $R_3$ . If the total resistance of the circuit is  $100 \Omega$ , determine the current flowing through resistor  $R_1$ . Find also the value of resistor  $R_2$ .



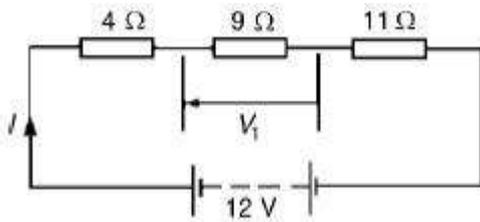
P.d. across R<sub>3</sub>,  $V_3 = 25 - 10 - 4 = 11V$  Current  $I = V/R$

$$= 25/100$$

$= 0.25A$ , which is the current flowing in each resistor Resistance  $R_2 = V_2/I$

$$= 4/0.25 = 16 \Omega$$

**Problem 3:** A 12V battery is connected in a circuit having three series-connected resistors having resistances of 4  $\Omega$ , 9  $\Omega$  and 11  $\Omega$ . Determine the current flowing through, and the p.d. across the 9  $\Omega$  resistor. Find also the power dissipated in the 11  $\Omega$  resistor.



Total resistance  $R = 4 + 9 + 11 = 24 \Omega$  Current  $I = V/R$

$$= 12/24$$

$= 0.5A$ , which is the current in the 9  $\Omega$  resistor. P.d. across the 9\_ resistor,  $V_1 = I \times 9 = 0.5 \times 9$

$$= 4.5V$$

Power dissipated in the 11  $\Omega$  resistor,  $P = I^2R = 0.5^2(11)$

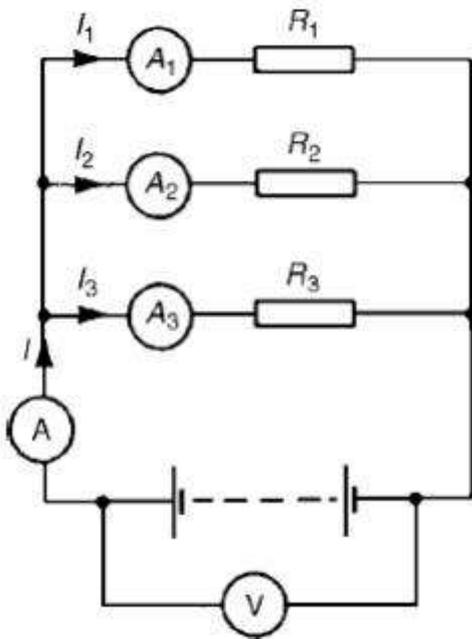
$$= 0.25(11)$$

$$= 2.75W$$

## 8. PARALLEL NETWORKS:

**Problem 1:** Figure shows three resistors, R<sub>1</sub>, R<sub>2</sub> and R<sub>3</sub> connected across each other, i.e. in parallel, across a battery source

of V volts.



**In a parallel circuit:**

(a) the sum of the currents  $I_1$ ,  $I_2$  and  $I_3$  is equal to the total circuit current,  $I$ , i.e.  $I = I_1 + I_2 + I_3$ , and

the source p.d.,  $V$  volts, is the same across each of the

From Ohm's law:

$$I_1 = V/R_1$$

$$, I_2 = V/R_2$$

$$, I_3 = V/R_3 \text{ and } I = V/R$$

where  $R$  is the total circuit resistance. Since  $I = I_1 + I_2 + I_3$

then

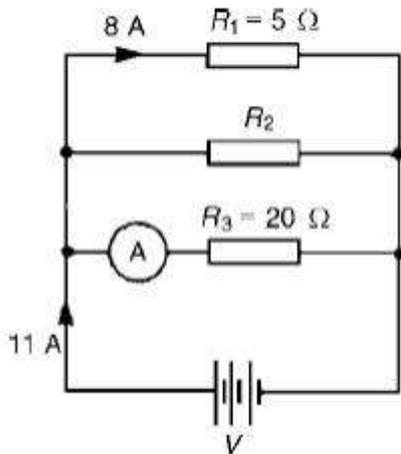
$V/R = V/R_1 + V/R_2 + V/R_3$  Dividing throughout by  $V$  gives:

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

This equation must be used when finding the total resistance  $R$  of a parallel circuit. For the special case of two resistors in parallel

$$\boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$$

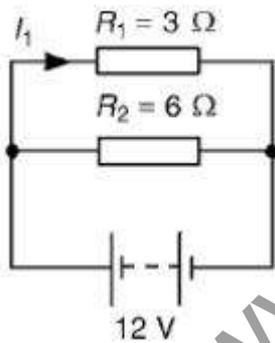
**Problem 2:** For the circuit shown in Figure , determine (a) the reading on the ammeter, and (b) the value of resistor R2.



P.d. across R1 is the same as the supply voltage V.  
Hence supply voltage,  $V = 8 \times 5 = 40\text{V}$

(a) Reading on ammeter,  $I = \frac{V}{R_3} = \frac{40}{20} = 2\text{A}$

Current flowing through R2  $= 11 - 8 - 2 = 1\text{A}$   
Hence,  $R_2 = \frac{V}{I_2} = \frac{40}{1} = 40 \Omega$

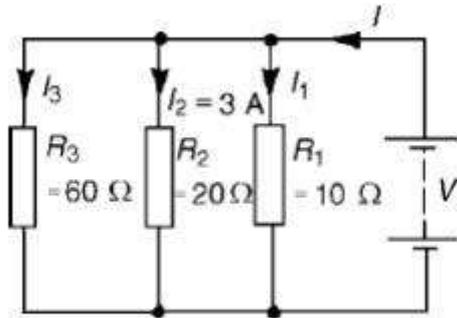


(a) The total circuit resistance R is given by  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{3} + \frac{1}{6}$

$\frac{1}{R} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6}$  Hence,  $R = \frac{6}{3} = 2 \Omega$

(b) Current in the 3 Ω resistance,  $I_1 = \frac{V}{R_1} = \frac{12}{3} = 4\text{A}$

**Problem 3:** For the circuit shown in Figure find (a) the value of the supply voltage V and (b) the value of current I.



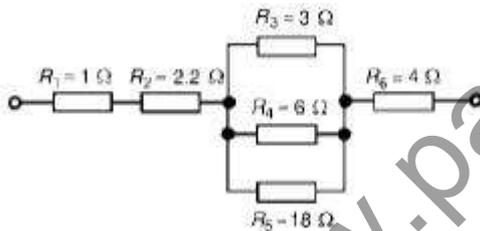
(a) P.d. across  $20\ \Omega$  resistor =  $I_2 R_2 = 3 \times 20 = 60\text{V}$ , hence supply voltage  $V = 60\text{V}$  since the circuit is connected in parallel.

(b) Current  $I_1 = V/R_1 = 60/10 = 6\text{A}$ ;  $I_2 = 3\text{A}$   
 $I_3 = V/R_3 = 60/60 = 1\text{A}$

Current  $I = I_1 + I_2 + I_3$  and hence  $I = 6 + 3 + 1 = 10\text{A}$  Alternatively,

$1/R = 1/60 + 1/20 + 1/10 = 1 + 3 + 6/60 = 10/60$  Hence total resistance  $R = 60/10 = 6\ \Omega$   
 Current  $I = V/R = 60/6 = 10\text{A}$

**Problem 4:** Find the equivalent resistance for the circuit shown in Figure



$R_3$ ,  $R_4$  and  $R_5$  are connected in parallel and their equivalent resistance  $R$  is given by:  $1/R = 1/3 + 1/6 + 1/18 = 6 + 3 + 1/18 = 10/18$

Hence  $R = 18/10 = 1.8\ \Omega$

The circuit is now equivalent to four resistors in series and the equivalent circuit resistance =  $1 + 2.2 + 1.8 + 4 = 9\ \Omega$

## 9. MESH ANALYSIS:

This is an alternative structured approach to solving the circuit and is based on calculating mesh currents. A similar approach to the node situation is used. A set of equations (based on KVL for each mesh) is formed and the equations are solved for unknown values. As many equations are needed as unknown mesh currents exist.

Step 1: Identify the mesh currents

Step 2: Determine which mesh currents are known

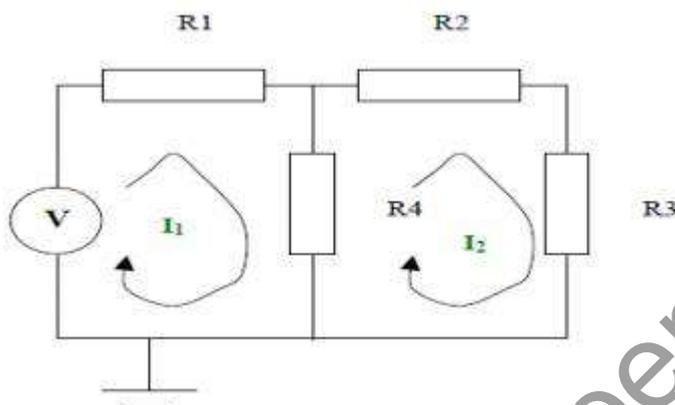
Step 2: Write equation for each mesh using KVL and that includes the mesh currents  
 Step 3: Solve the equations

**Step 1:**

The mesh currents are as shown in the diagram on the next page

**Step 2:**

Neither of the mesh currents is known



**Step 3:**

KVL can be applied to the left hand side loop. This states the voltages around the loop sum to zero.

When writing down the voltages across each resistor equations are the mesh currents.

$$I_1 R_1 + (I_1 - I_2) R_4 - V = 0$$

KVL can be applied to the right hand side loop. This states the voltages around the loop sum to

zero. When writing down the voltages across ea the equations are the mesh currents.

$$I_2 R_2 + I_2 R_3 + (I_2 - I_1) R_4 = 0$$

**Step 4:**

Solving the equations we get

$$I_1 = V \frac{R_2 + R_3 + R_4}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_4 + R_3 R_4}$$

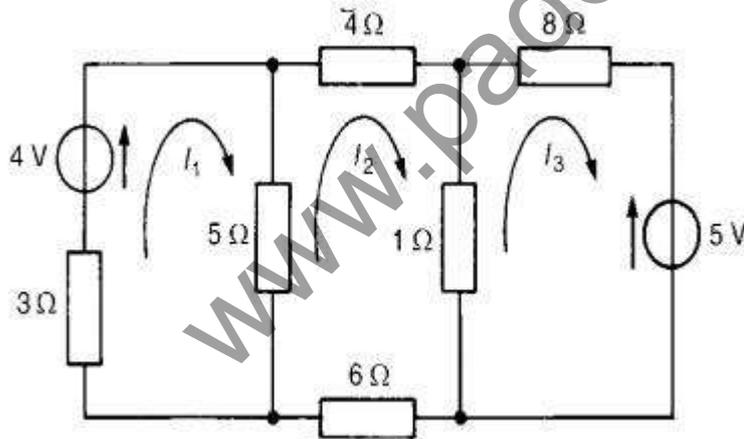
$$I_2 = V \frac{R_4}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_4 + R_3 R_4}$$

The individual branch currents can be obtained from these mesh currents and the node voltages can also be calculated using this information. For example:

$$V_C = I_2 R_3 = V \frac{R_3 R_4}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_4 + R_3 R_4}$$

**Problem 1:**

Use mesh-current analysis to determine the current flowing in (a) the 1Ω resistance of the d.c. circuit shown in



The mesh currents I1, I2 and I3 are shown in Figure

Using Kirchhoff's voltage law:

For loop 1,  $(3 + 5) I_1 - I_2 = 4$  .....(1)

For loop 2,  $(4 + 1 + 6 + 5) I_2 - (5) I_1 - (1) I_3 = 0$  .....(2)

For loop 3,  $(1 + 8) I_3 - (1) I_2 = 5$  .....(3)

Thus

$$8I_1 - 5I_2 - 4 = 0$$

$$-5I_1 + 16I_2 - I_3 = 0$$

$$-I_2 + 9I_3 + 5 = 0$$

$$\frac{I_1}{\begin{vmatrix} -5 & 0 & -4 \\ 16 & -1 & 0 \\ -1 & 9 & 5 \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} 8 & 0 & -4 \\ -5 & -1 & 0 \\ 0 & 9 & 5 \end{vmatrix}} = \frac{I_3}{\begin{vmatrix} 8 & -5 & -4 \\ -5 & 16 & 0 \\ 0 & -1 & 5 \end{vmatrix}}$$

$$= \frac{-1}{\begin{vmatrix} 8 & -5 & 0 \\ -5 & 16 & -1 \\ 0 & -1 & 9 \end{vmatrix}}$$

Using determinants,

$$\frac{I_1}{-5 \begin{vmatrix} -1 & 0 \\ 9 & 5 \end{vmatrix} - 4 \begin{vmatrix} 16 & -1 \\ -1 & 9 \end{vmatrix}} = \frac{-I_2}{8 \begin{vmatrix} -1 & 0 \\ 9 & 5 \end{vmatrix} - 4 \begin{vmatrix} -5 & -1 \\ 0 & 9 \end{vmatrix}}$$

$$= \frac{I_3}{-4 \begin{vmatrix} -5 & 16 \\ 0 & -1 \end{vmatrix} + 5 \begin{vmatrix} 8 & -5 \\ -5 & 16 \end{vmatrix}}$$

$$= \frac{-1}{8 \begin{vmatrix} 16 & -1 \\ -1 & 9 \end{vmatrix} + 5 \begin{vmatrix} -5 & -1 \\ 0 & 9 \end{vmatrix}}$$

$$\frac{I_1}{-5(-5) - 4(143)} = \frac{-I_2}{8(-5) - 4(-45)}$$

$$= \frac{I_3}{-4(5) + 5(103)}$$

$$\begin{aligned} &= \frac{I_3}{-4(5) + 5(103)} \\ &= \frac{-1}{8(143) + 5(-45)} \\ \frac{I_1}{-547} &= \frac{-I_2}{140} = \frac{I_3}{495} = \frac{-1}{919} \end{aligned}$$

Hence  $I_1 = \frac{547}{919} = 0.595 \text{ A}$ ,

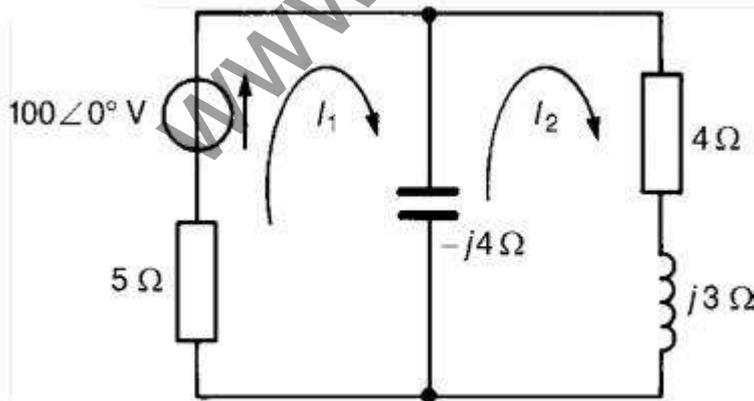
$I_2 = \frac{140}{919} = 0.152 \text{ A}$ , and

$I_3 = \frac{-495}{919} = -0.539 \text{ A}$

(a) Current in the  $5 \Omega$  resistance =  $I_1 - I_2$   
 $= 0.595 - 0.152$   
 $= 0.44 \text{ A}$

(b) Current in the  $1 \Omega$  resistance =  $I_2 - I_3$   
 $= 0.152 - (-0.539)$   
 $= 0.69 \text{ A}$

**Problem 2:** For the a.c. network shown in Figure determine, using mesh-current analysis, (a) the mesh currents  $I_1$  and  $I_2$  (b) the current flowing in the capacitor, and (c) the active power delivered by the  $100 \angle 0^\circ \text{ V}$  voltage source.



(a) For the first loop  
 $(5 - j4) I_1 - (-j4) I_2 = 100 \angle 0^\circ$ .....(1) For the second loop

$$(4+j3-j4)I_2 = 0 \dots\dots\dots (2) \quad -(-j4I_1)$$

Rewriting equations (1) and (2) gives:

$$(5 -j4)I_1 + j4I_2 -100 =0$$

$j4I_1 + (4 -j) I_2 + 0 =0$  Thus, using determinants,

(b) Current flowing

$$\begin{aligned} &= I_1 - I_2 \\ &= 10.77 \angle 19. \\ &= 4.44 + j12 \end{aligned}$$

$$\frac{I_1}{\begin{vmatrix} j4 & -100 \\ (4-j) & 0 \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} (5-j4) & -100 \\ j4 & 0 \end{vmatrix}}$$

$$= \frac{1}{\begin{vmatrix} (5-j4) & j4 \\ j4 & (4-j) \end{vmatrix}}$$

i.e. the current

(c) Source power  $P$

$$\frac{I_1}{(400 -j100)} = \frac{-I_2}{j400} = \frac{1}{(32 -j21)}$$

$$\begin{aligned} \text{Hence } I_1 &= \frac{(400 -j100)}{(32 -j21)} = \frac{412.31 \angle -14.04^\circ}{38.28 \angle -33.27^\circ} \\ &= 10.77 \angle 19.23^\circ \text{ A} = 10.8 \angle -19.2^\circ \text{ A,} \end{aligned}$$

(Check: power i

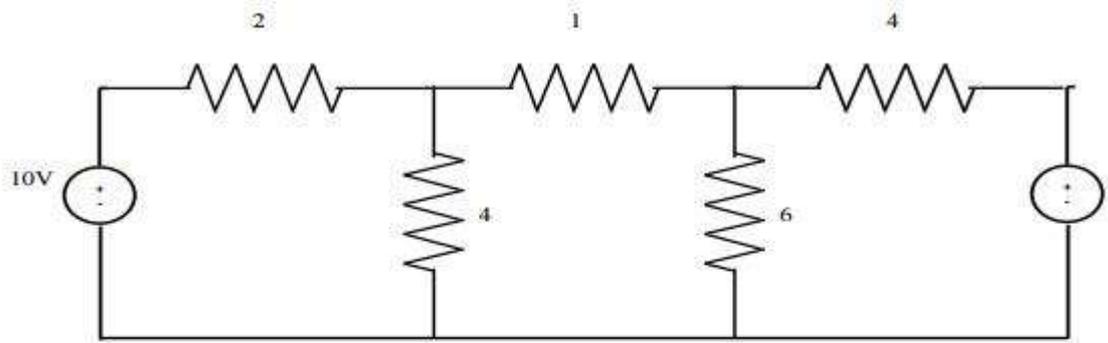
correct to one decimal place

anc

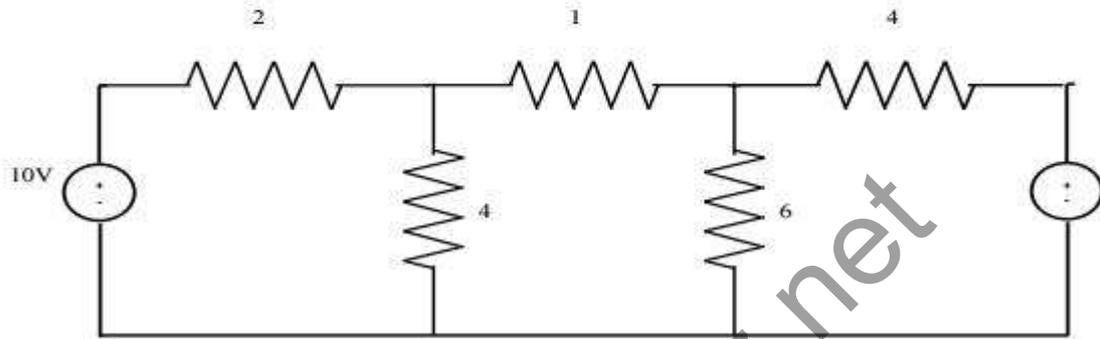
$$\begin{aligned} I_2 &= \frac{400 \angle -90^\circ}{38.28 \angle -33.27^\circ} = 10.45 \angle -56.73^\circ \text{ A} \\ &= 10.5 \angle -56.7^\circ \text{ A,} \end{aligned}$$

Thus total power dissipated = 579.97 + 436.81 = 1016.8W = 1020W

**Problem 3:** Calculate current through  $-6\Omega$  resistance u



**Solution:**



Case(1): Consider loop ABGH ; Apply KVL .

Case(1): Consider loop ABGH ; Apply KVL .

$$10 = 2I_1 + 4(I_1 - I_2)$$

$$10 = 6I_1 - 4I_2 \text{ ----- (1)}$$

Consider loop BCFG

$$I_2 + 6(I_2 + I_3) + 4(I_2 - I_1) = 0$$

$$11I_2 + 6I_3 - 4I_1 = 0 \text{ ----- (2)}$$

Consider loop CDEF

$$20 = 4I_3 + 6(I_2 - I_3)$$

$$20 = 10I_3 + 6I_2 \text{ ----- (3)}$$

$$D = \begin{vmatrix} 6 & -4 & 0 \\ -4 & 11 & 6 \\ 0 & 6 & 10 \end{vmatrix}$$

$$= \begin{vmatrix} 10 \\ 0 \\ 20 \end{vmatrix}$$

$$D = [6(110 - 36) + 4(-40)] = 284.$$

$$D_1 = \begin{vmatrix} 10 & -4 & 0 \\ 0 & 11 & 6 \\ 20 & 6 & 10 \end{vmatrix}$$

$$D_1 = 10[110 - 36 + (-120)]$$

$$= 260$$

$$D_2 = \begin{vmatrix} 6 & 10 & 0 \\ -4 & 0 & 6 \\ 0 & 20 & 10 \end{vmatrix}$$

$$D_2 = 6(-120) - 10(-40) = -320$$

$$D_3 = \begin{vmatrix} 6 & -4 & 10 \\ -4 & 11 & 0 \\ 0 & 6 & 20 \end{vmatrix}$$

$$D_3 = 6(220) + 4(-80) + 10(-24)$$

$$D_3 = 760$$

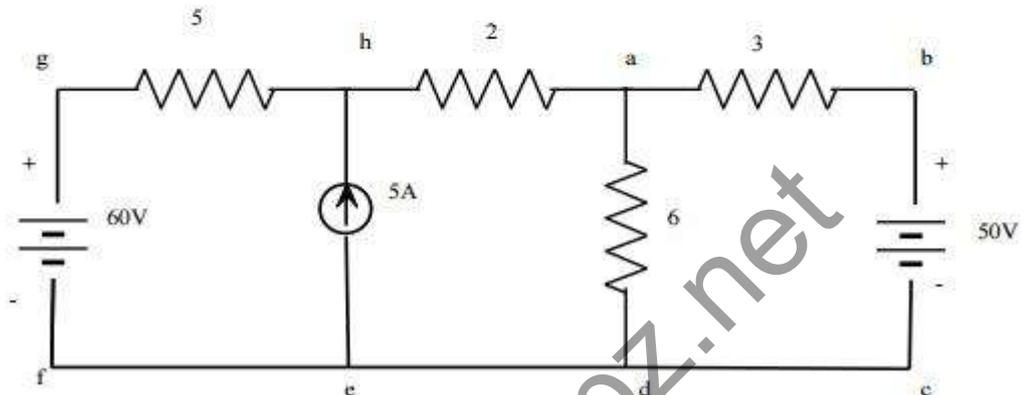
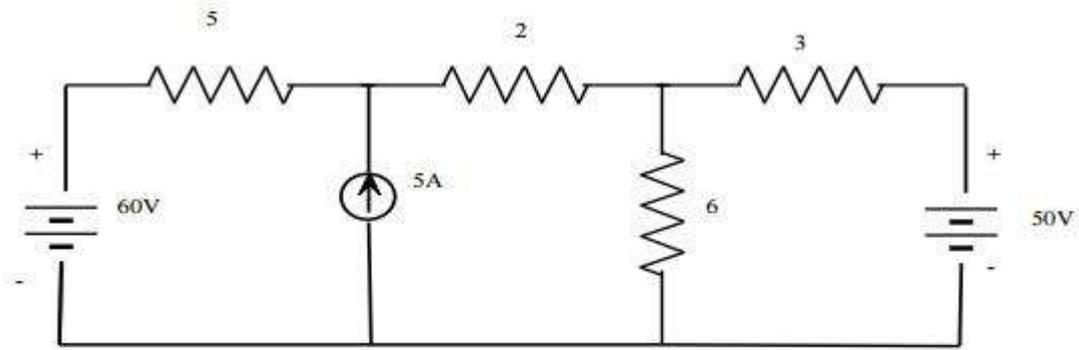
$$I_1 = D_1/D = 260/284 = 0.915A$$

$$I_2 = D_2/D = -320/284 = -1.1267A \quad I_3 = D_3/D = 760/284 = 2.676A$$

$$\text{Current through } 6\Omega \text{ 2 +resistance3} = I$$

$$= -1.1267 + 2.676 = 1.55A$$

**Problem 4:** Find the current through branch a-b using mesh analysis.



Solution:

Consider loops

Loop HADE  $\rightarrow 5I_1 + 2I_2 + 6(I_2 - I_3) = 60$

$$5I_1 + 8I_2 - 6I_3 = 60 \text{ ----- (1)}$$

Loop ABCDA  $\rightarrow 3I_3 + 6(I_3 - I_2) = -50$

$$3I_3 + 6I_3 - 6I_2 = -50$$

$$9I_3 - 6I_2 = -50 \text{ ----- (2)}$$

$$I_2 - I_1 = 5A \text{ ----- (3)}$$

From (1), (2) & (3).

$$D = \begin{vmatrix} -1 & 1 & 0 \\ 5 & 8 & -6 \\ 0 & -6 & 9 \end{vmatrix}$$

$$= -1(72-36) - 1(45)$$

$$D = -81$$

$$D_3 = \begin{vmatrix} -1 & 1 & 5 \\ 5 & 8 & 60 \\ 0 & -6 & -50 \end{vmatrix}$$

$$= -1(-400+360) - (-250) + 5(-30)$$

$$\begin{aligned} &= 40+250-150 \\ D3 &= 140. \\ I3 &= D3/D = 140/-81 = -1.7283 \end{aligned}$$

The current through branch ab is 1.7283A which is flowing from b to a.

## 10. NODAL ANALYSIS:

Nodal analysis involves looking at a circuit and determining all the node voltages in the circuit. The voltage at any given node of a circuit is the voltage drop between that node and a reference node (usually ground). Once the node voltages are known any of the

currents flowing in the circuit can be determined. The node method offers an organized way of achieving this.

### Approach:

Firstly all the nodes in the circuit are counted and identified. Secondly nodes at which the voltage is already known are listed. A set of equations based on the node voltages are formed and these equations are solved for unknown quantities. The set of equations are formed using KCL at each node. The set of simultaneous equations that is produced is then solved. Branch currents can then be found once the node voltages are known. This can be reduced to a series of steps:

Step 1: Identify the nodes

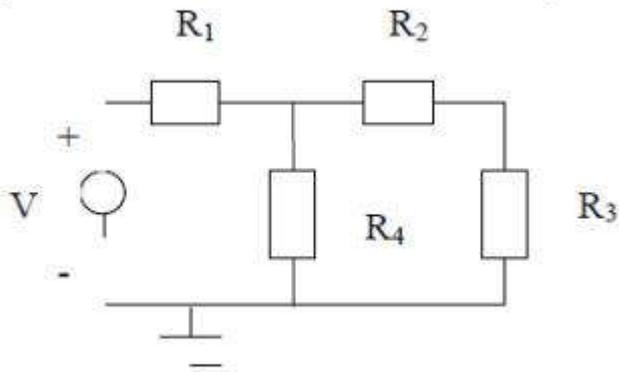
Step 2: Choose a reference node

Step 3: Identify which node voltages are known if any Step 4: Identify the BRANCH currents

Step 5: Use KCL to write an equation for each unknown node voltage Step

6: Solve the equations

This is best illustrated with an example. Find all currents and voltages in the following circuit using the node method. (In this particular case it can be solved in other ways as well)



**Step 1:**

There are four nodes in the circuit. A, B, C and D

**Step 2:**

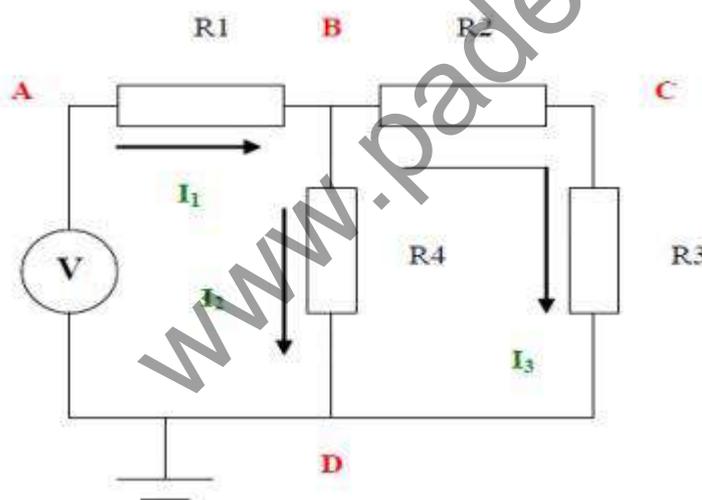
Ground, node D is the reference node.

**Step 3:**

Node voltage B and C are unknown. Voltage at A is V and at D is 0

**Step 4:**

The currents are as shown. There are 3 different currents



**Step 5:**

I need to create two equations so I apply KCL at node B and node C

The statement of KCL for node B is as follows:

$$\frac{V - V_B}{R_1} + \frac{V_C - V_B}{R_2} + \frac{-V_B}{R_4} = 0$$

The statement of KCL for node C is as follows:

$$\frac{V_C - V_B}{R_2} + \frac{-V_B}{R_3} = 0$$

**Step 6:**

We now have two equations to solve for the two unknowns  $V_B$  and  $V_C$ . Solving the above two equations we get:

$$V_C = V \frac{R_3 R_4}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_4 + R_3 R_4}$$

$$V_B = V \frac{R_4 (R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_4 + R_3 R_4}$$

**Further Calculations**

The node voltages are now all known. From these we can get the branch currents by a simple application of Ohm's Law:

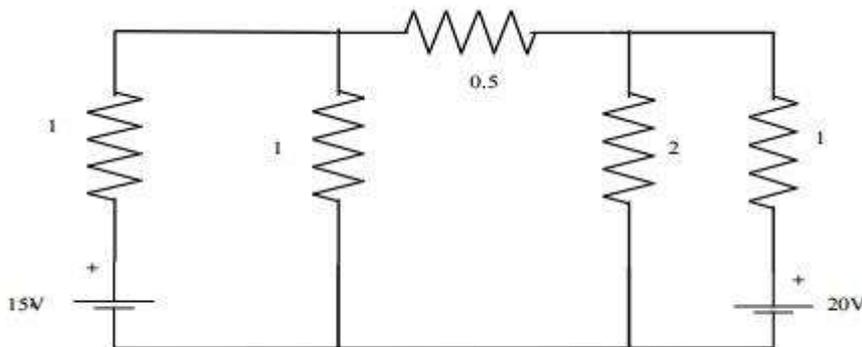
$$I_1 = (V - V_B) / R_1$$

$$I_2 = (V_B - V_C) / R_2$$

$$I_3 = (V_C) / R_3$$

$$I_4 = (V_B) / R_4$$

**Problem 1:** Find the current through each resistor of the circuit shown in fig, using nodal analysis



**Solution:**

**At node1,**

$$-I_1 - I_2 - I_3 = 0 \quad -[V_1 - 15/1] - [V_1/1] - [V_1 - V_2/0.5] = 0$$

$$-V_1 + 15 - V_1 - 2V_1 + 2V_2 = 0$$

$$4V_1 - 2V_2 = 15 \quad \text{----- (1)}$$

**At node2,**

$$I_3 - I_4 - I_5 = 0$$

$$V_1 - V_2/0.5 - V_2/2 - V_2 - 20/1 = 0$$

$$2V_1 - 2V_2 - 0.5V_2 - V_2 + 20 = 0$$

$$2V_1 - 3.5V_2 = -20 \quad \text{----- (2)}$$

Multiplying (2) by 2 & subtracting from (1)

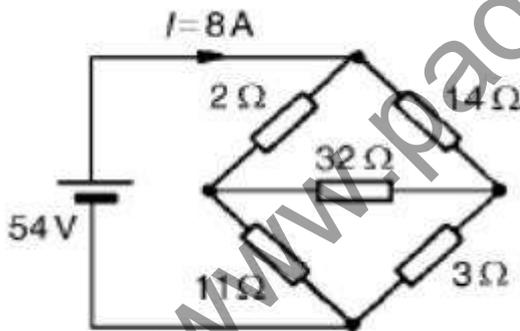
$$5V_2 = 55 \quad V_2 = 11V \quad V = 9.25V$$

$$I_1 = V_1 - 15/1 = 9.25 - 15 = -5.75A = 5.75A \quad I_2 = V_1/1 = 9.25A$$

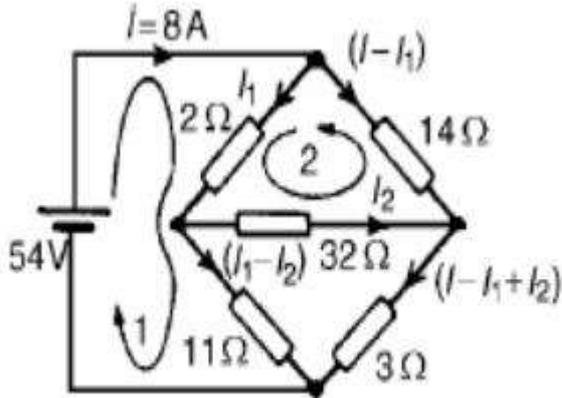
$$I_3 = V_1 - V_2/0.5 = -3.5A = 3.5A \quad \leftarrow I_4 = V_2/2 = 5.5A$$

$$I_5 = V_2 - 20/1 = 11 - 20/1 = -9A = 9A.$$

**Problem 2:** For the bridge network shown in Figure determine the currents in each of the resistors.



Let the current in the  $2\Omega$  resistor be  $I_1$ , and then the current by law, Kirchhoff's current in the  $14\Omega$  resistor is  $(I - I_1)$ . Let the current in the  $32\Omega$  resistor be  $I_2$  as shown in Figure. Then the current in the  $1\Omega$  resistor is  $(I_1 - I_2)$  and that in the  $3\Omega$  resistor is  $(I - I_1 + I_2)$ . Applying Kirchhoff's and moving in a clockwise direction as shown in Figure gives:



$$54 = 2I_1 + 11(I_1 - I_2)$$

i.e.  $13I_1 - 11I_2 = 54$

Applying Kirchhoff's voltage law to loop 2 and direction as shown in Figure gives:

$$0 = 2I_1 + 32I_2 - 14(I - I_1)$$

However  $I = 8 \text{ A}$

Hence  $0 = 2I_1 + 32I_2 - 14(8 - I_1)$  i.e.  $16I_1 + 32I_2 = 112$

Equations (1) and (2) are simultaneous equations with two unknowns,  $I_1$  and  $I_2$ .

$16 * (1)$  gives:  $208I_1 - 176I_2 = 864$

$16 * (2)$  gives:  $208I_1 - 176I_2 = 864$

$13 * (2)$  gives:  $208I_1 + 416I_2 = 1456$

$(4) - (3)$  gives:  $592I_2 = 592, I_2 = 1 \text{ A}$

Substituting for  $I_2$  in (1) gives:

$$13I_1 - 11 = 54$$

$$I_1 = 65/13 = 5 \text{ A}$$

Hence,

the current flowing in the  $2 \Omega$  resistor =  $I_1 = 5 \text{ A}$

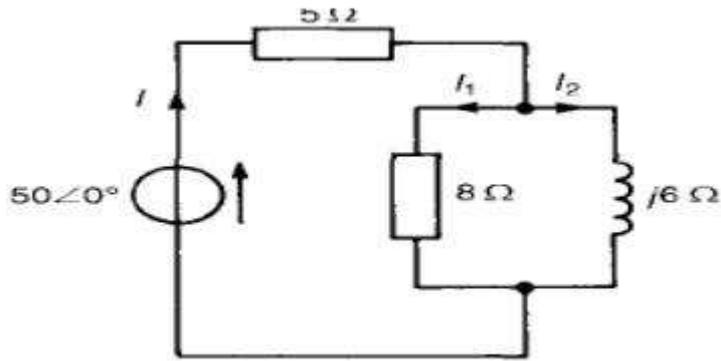
the current flowing in the  $14 \Omega$  resistor =  $I - I_1 = 8 - 5 = 3 \text{ A}$

the current flowing in the  $32 \Omega$  resistor =  $I_2 = 1 \text{ A}$

the current flowing in the  $11 \Omega$  resistor =  $I_1 - I_2 = 5 - 1 = 4 \text{ A}$  and

the current flowing in the  $3 \Omega$  resistor =  $I - I_1 + I_2 = 8 - 5 + 1 = 4 \text{ A}$

Problem 3: Determine the values of currents  $I$ ,  $I_1$  and  $I_2$  shown in the network of Figure



Total circuit impedance,

$$\begin{aligned} Z_T &= 5 + (8)(j6)/8 + j6 \\ &= 5 + (j48)(8 - j6)/82 + 62 \\ &= 5 + (j384 + 288)/100 \end{aligned}$$

$$= (7.88 + j3.84) \text{ or } 8.776 \angle 25.98^\circ \text{ A Current } I = V/Z_T$$

$$\begin{aligned} &= \frac{50 \angle 0^\circ}{8.77 \angle 25.98^\circ} \\ &= 5.7066 \angle -25.98^\circ \text{ A} \end{aligned}$$

Current  $I_1 = I (j6/8 + j6)$

$$\begin{aligned} &= \frac{(5.702 \angle -5.98^\circ) (6 \angle 90^\circ)}{10 \angle 36.87^\circ} \\ &= 3.426 \angle 27.15^\circ \text{ A} \end{aligned}$$

Current  $I_2 = I (8 / (8 + j6))$

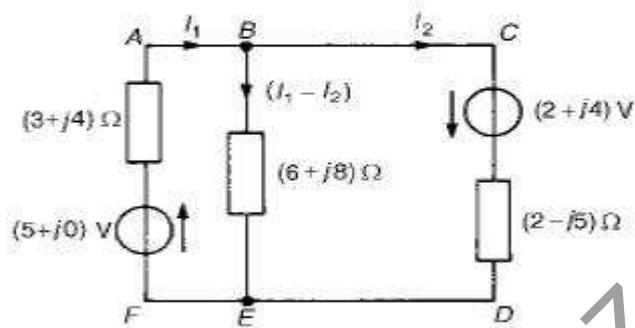
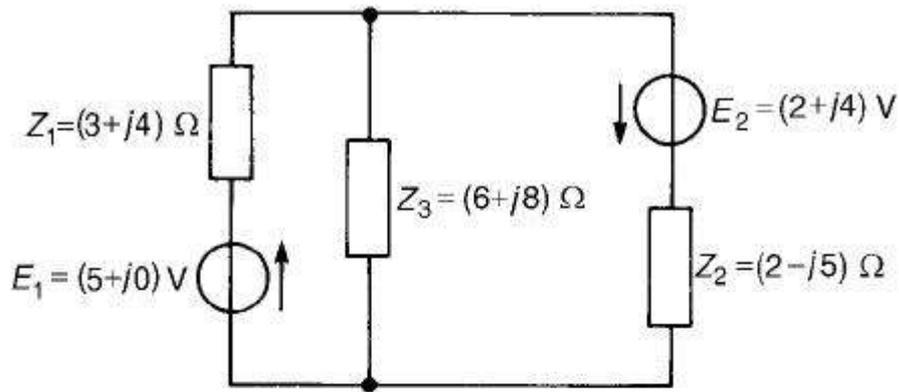
$$\begin{aligned} &= \frac{(5.2570 \angle -9.8^\circ) * 8 \angle 0^\circ}{10 \angle 36.87^\circ} \\ &= 4.5666 \angle -62.85^\circ \text{ A} \end{aligned}$$

[Note:  $I = I_1 + I_2 = 3.42 \angle 27.15^\circ + 4.56 \angle -62.85^\circ$

$$= 3.043 + j1.561 + 2.081 - j4.058$$

$$= 5.124 - j2.497 \text{ A} = 5.706 \angle -25.98^\circ \text{ A}$$

**Problem 4:** For the a.c. network shown in Figure, determine the current flowing in each branch using Kirchhoff's laws.



from which,  $I_1 = \frac{20 + j55}{64 + j27} = \frac{58.52 \angle 70.02^\circ}{69.46 \angle 22.87^\circ} = 0.842 \angle 47.15^\circ \text{ A}$   
 $= (0.573 + j0.617) \text{ A}$   
 $= (0.57 + j0.62) \text{ A, correct to two decimal places.}$

From equation (1),  $5 = (9 + j12)(0.573 + j0.617) - (6 + j8)I_2$

$$5 = (-2.247 + j12.429) - (6 + j8)I_2$$

from which,  $I_2 = \frac{-2.247 + j12.429 - 5}{6 + j8}$

$$= \frac{14.39 \angle 120.25^\circ}{10 \angle 53.13^\circ}$$

$$= 1.439 \angle 67.12^\circ \text{ A} = (0.559 + j1.326) \text{ A}$$

$$= (0.56 + j1.33) \text{ A, correct to two decimal places.}$$

The current in the  $(6 + j8)\Omega$  impedance,

$$I_1 - I_2 = (0.573 + j0.617) - (0.559 + j1.326)$$

$$= (0.014 - j0.709) \text{ A or } 0.709 \angle -88.87^\circ \text{ A}$$

An alternative method of solving equations (1) and (2) is shown below, using determinants.

$$(9 + j12)I_1 - (6 + j8)I_2 - 5 = 0 \tag{1}$$

$$-(6 + j8)I_1 + (8 + j3)I_2 - (2 + j4) = 0 \tag{2}$$

$$\text{Thus } \frac{I_1}{\begin{vmatrix} -(6+j8) & -5 \\ (8+j3) & -(2+j4) \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} (9+j12) & -5 \\ -(6+j8) & -(2+j4) \end{vmatrix}}$$

$$= \frac{1}{\begin{vmatrix} (9+j12) & -(6+j8) \\ -(6+j8) & (8+j3) \end{vmatrix}}$$

$$\frac{I_1}{(-20+j40) + (40+j15)} = \frac{-I_2}{(30-j60) - (30+j40)}$$

$$= \frac{1}{(36+j123) - (-28+j96)}$$

$$\frac{I_1}{20+j55} = \frac{-I_2}{-j100} = \frac{1}{64+j27}$$

$$\text{Hence } I_1 = \frac{20+j55}{64+j27} = \frac{58.52\angle 70.02^\circ}{69.46\angle 22.87^\circ}$$

$$= 0.842\angle 47.15^\circ \text{ A}$$

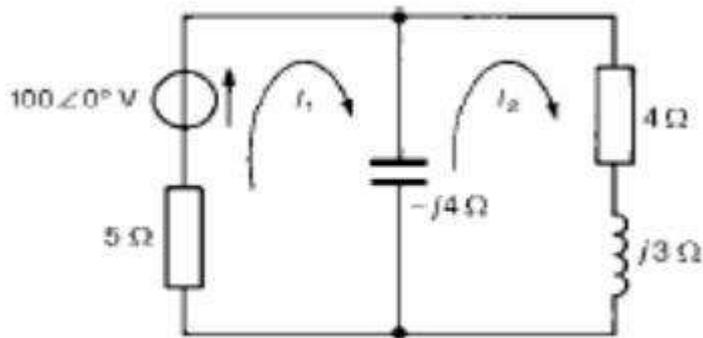
$$\text{and } I_2 = \frac{100\angle 90^\circ}{69.46\angle 22.87^\circ} = 1.440\angle 67.13^\circ \text{ A}$$

The current flowing in the  $(6+j8) \Omega$  impedance is given by:

$$I_1 - I_2 = 0.842\angle 47.15^\circ - 1.440\angle 67.13^\circ \text{ A}$$

$$= (0.013 - j0.709) \text{ A or } 0.709\angle -88.95^\circ \text{ A}$$

**Problem 5:** For the a.c. network shown in Figure determine, using mesh-current analysis, (a) the mesh currents  $I_1$  and  $I_2$  (b) the current flowing in the capacitor, and (c) the active power delivered by the  $100\angle 0^\circ$  V voltage source.



(a) For the first loop  $(5 - j4)I_1 - (-j4I_2) = 100\angle 0^\circ$  (1)

For the second loop  $(4 + j3 - j4)I_2 - (-j4I_1) = 0$  (2)

Rewriting equations (1) and (2) gives:

$$(5 - j4)I_1 + j4I_2 - 100 = 0 \quad (1')$$

$$j4I_1 + (4 - j)I_2 + 0 = 0 \quad (2')$$

Thus, using determinants,

$$\frac{I_1}{\begin{vmatrix} j4 & -100 \\ (4 - j) & 0 \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} (5 - j4) & -100 \\ j4 & 0 \end{vmatrix}} = \frac{1}{\begin{vmatrix} (5 - j4) & j4 \\ j4 & (4 - j) \end{vmatrix}}$$

$$\frac{I_1}{(400 - j100)} = \frac{-I_2}{j400} = \frac{1}{(32 - j21)}$$

Hence  $I_1 = \frac{(400 - j100)}{(32 - j21)} = \frac{412.31\angle -14.04^\circ}{38.28\angle -33.27^\circ}$

$$= 10.77\angle 19.23^\circ \text{ A} = 10.8\angle -19.2^\circ \text{ A,}$$

correct to one decimal place

$$I_2 = \frac{400\angle -90^\circ}{38.28\angle -33.27^\circ} = 10.45\angle -56.73^\circ \text{ A}$$

$$= 10.5\angle -56.7^\circ \text{ A,}$$

correct to one decimal place

(b) Current flowing in capacitor  $= I_1 - I_2$

$$= 10.77 \angle 19.23^\circ - 10.45 \angle -56.73^\circ$$

$$= 4.44 + j12.28 = 13.1 \angle 70.12^\circ \text{ A,}$$

i.e., the current in the capacitor is 13.1 A

(c) Source power  $P = VI \cos \phi = (100)(10.77) \cos 19.23^\circ$

$$= 1016.9 \text{ W} = 1020 \text{ W,}$$

correct to three significant figures.

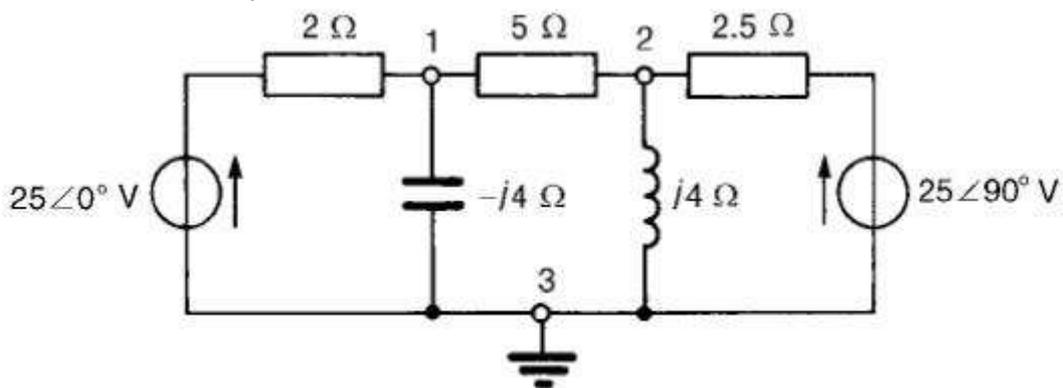
(Check: power in  $5 \Omega$  resistor  $= I_1^2(5) = (10.77)^2(5) = 579.97 \text{ W}$

and power in  $4 \Omega$  resistor  $= I_2^2(4) = (10.45)^2(4) = 436.81 \text{ W}$

Thus total power dissipated  $= 579.97 + 436.81$

$= 1016.8 \text{ W} = 1020 \text{ W, correct to three significant figures.})$

**Problem 6:** In the network of Figure use nodal analysis to determine (a) the voltage at nodes 1 and 2, (b) the current in the  $j4 \Omega$  inductance, (c) of the active power dissipated-10) in the  $2.5 \Omega$



(a) At node 1,  $\frac{V_1 - 25\angle 0^\circ}{2} + \frac{V_1}{-j4} + \frac{V_1 - V_2}{5} = 0$

Rearranging gives:

$$\left(\frac{1}{2} + \frac{1}{-j4} + \frac{1}{5}\right)V_1 - \left(\frac{1}{5}\right)V_2 - \frac{25\angle 0^\circ}{2} = 0$$

i.e.,  $(0.7 + j0.25)V_1 - 0.2V_2 - 12.5 = 0$  (1)

At node 2,  $\frac{V_2 - 25\angle 90^\circ}{2.5} + \frac{V_2}{j4} + \frac{V_2 - V_1}{5} = 0$

Rearranging gives:

$$-\left(\frac{1}{5}\right)V_1 + \left(\frac{1}{2.5} + \frac{1}{j4} + \frac{1}{5}\right)V_2 - \frac{25\angle 90^\circ}{2.5} = 0$$

i.e.,  $-0.2V_1 + (0.6 - j0.25)V_2 - j10 = 0$  (2)

Thus two simultaneous equations have been formed with two unknowns,  $V_1$  and  $V_2$ . Using determinants, if

$$(0.7 + j0.25)V_1 - 0.2V_2 - 12.5 = 0$$
 (1)

and  $-0.2V_1 + (0.6 - j0.25)V_2 - j10 = 0$  (2)

then

$$\begin{aligned} \frac{V_1}{\begin{vmatrix} -0.2 & -12.5 \\ (0.6 - j0.25) & -j10 \end{vmatrix}} &= \frac{-V_2}{\begin{vmatrix} (0.7 + j0.25) & -12.5 \\ -0.2 & -j10 \end{vmatrix}} \\ &= \frac{1}{\begin{vmatrix} (0.7 + j0.25) & -0.2 \\ -0.2 & (0.6 - j0.25) \end{vmatrix}} \end{aligned}$$

i.e.,

$$\frac{V_1}{(j2 + 7.5 - j3.125)} = \frac{-V_2}{(-j7 + 2.5 - 2.5)}$$

$$= \frac{1}{(0.42 - j0.175 + j0.15 + 0.0625 - 0.04)}$$

and  $\frac{V_1}{7.584\angle-8.53^\circ} = \frac{-V_2}{-7\angle90^\circ} = \frac{1}{0.443\angle-3.23^\circ}$

Thus voltage,  $V_1 = \frac{7.584\angle-8.53^\circ}{0.443\angle-3.23^\circ} = 17.12\angle-5.30^\circ \text{ V}$

$= 17.1\angle-5.3^\circ \text{ V}$ , correct to one decimal place,

and voltage,  $V_2 = \frac{7\angle90^\circ}{0.443\angle-3.23^\circ} = 15.80\angle93.23^\circ \text{ V}$

$= 15.8\angle93.2^\circ \text{ V}$ , correct to one decimal place.

(b) The current in the  $j4 \Omega$  inductance is given by:

$$\frac{V_2}{j4} = \frac{15.80\angle93.23^\circ}{4\angle90^\circ} = 3.95\angle3.23^\circ \text{ A flowing away from node 2}$$

(c) The current in the 5 Ω resistance is given by:

$$I_5 = \frac{V_1 - V_2}{5} = \frac{17.12\angle-5.30^\circ - 15.80\angle93.23^\circ}{5}$$

$$\text{i.e., } I_5 = \frac{(17.05 - j1.58) - (-0.89 + j15.77)}{5}$$

$$= \frac{17.94 - j17.35}{5} = \frac{24.96\angle-44.04^\circ}{5}$$

$$= 4.99\angle-44.04^\circ \text{ A flowing from node 1 to node 2}$$

(d) The active power dissipated in the 2.5 Ω resistor is given by

$$P_{2.5} = (I_{2.5})^2(2.5) = \left(\frac{V_2 - 25\angle90^\circ}{2.5}\right)^2 (2.5)$$

$$= \frac{(0.89 + j15.77 - j25)^2}{2.5} = \frac{(9.273\angle-95.51^\circ)^2}{2.5}$$

$$= \frac{85.99\angle-191.02^\circ}{2.5} \text{ by de Moivre's theorem}$$

$$= 34.4\angle169^\circ \text{ W}$$

## BASIC ELEMENTS & INTRODUCTORY CONCEPTS:

### Electrical Network:

A combination of various electric elements (Resistor, Inductor, Capacitor, Voltage source, Current source) connected in any manner what so ever is called an electrical network. We may classify circuit elements in two categories, passive and active elements.

### Passive Element:

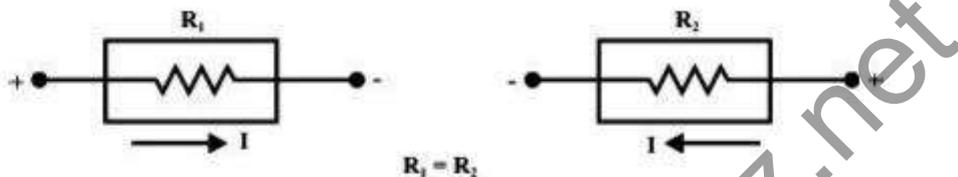
The element which receives energy (or absorbs energy) and then either converts it into heat (R) or stored it in an electric (C) or magnetic (L) field is called passive element.

### Active Element:

The elements that supply energy to the circuit is called active element. Examples of active elements include voltage and current sources, generators, and electronic devices that require power supplies. A transistor is an active circuit element, meaning that it can amplify power of a signal. On the other hand, transformer is not an active element because it does not amplify the power level and power remains same both in primary and secondary sides. Transformer is an example of passive element.

### Bilateral Element:

Conduction of current in both directions in an element (example: Resistance; Inductance; Capacitance) with same magnitude is termed as bilateral element.



### Unilateral Element:

Conduction of current in one direction is termed as unilateral (example: Diode, Transistor) element.



### Meaning of Response:

An application of input signal to the system will produce an output signal, the behavior of output signal with time is known as the response of the system.

### Potential Energy Difference:

The voltage or potential energy difference between two points in an electric circuit is the amount of energy required to move a unit charge between the two points.

### Ohm's Law:

Ohm's law states that the current through a conductor between two points is directly proportional to the potential difference or voltage across the two points, and inversely proportional to the resistance between them. The mathematical equation that describes this relationship is:

$$I = V / R$$

$$I = \frac{V}{R}$$

where I is the current through the resistance in units of amperes, V is the potential difference measured across the resistance in units of volts, and R is the resistance of the conductor in units of ohms. More specifically, Ohm's law states that the R in this relation is constant, independent of the current.

## KIRCHOFF'S LAW

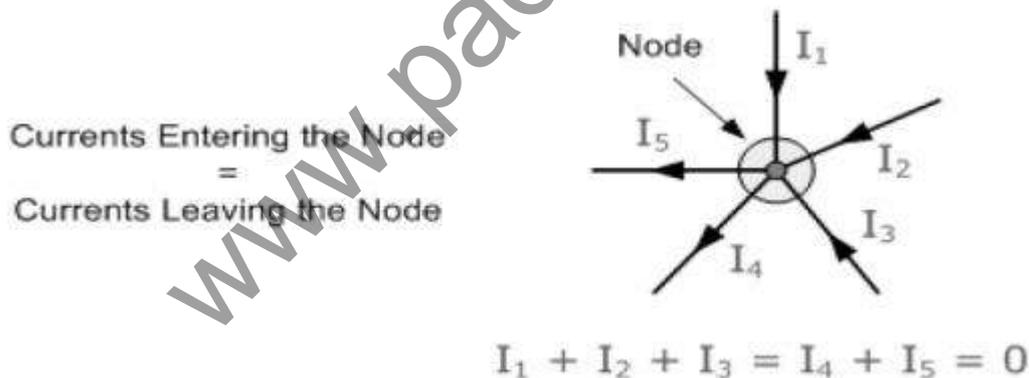
### Kirchoff's First Law - The Current Law, (KCL)

"The total current or charge entering a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node".

In other words the algebraic sum of ALL the currents entering and leaving a node must be equal to zero,

$$I(\text{exiting}) + I(\text{entering}) = 0.$$

This idea by Kirchoff is known as the Conservation of Charge.



Here, the 3 currents entering the node, I1, I2, I3 are all positive in value and the 2 currents leaving the node, I4 and I5 are negative in value.

Then this means we can also rewrite the equation as;  $I_1 + I_2 + I_3 - I_4 - I_5 = 0$

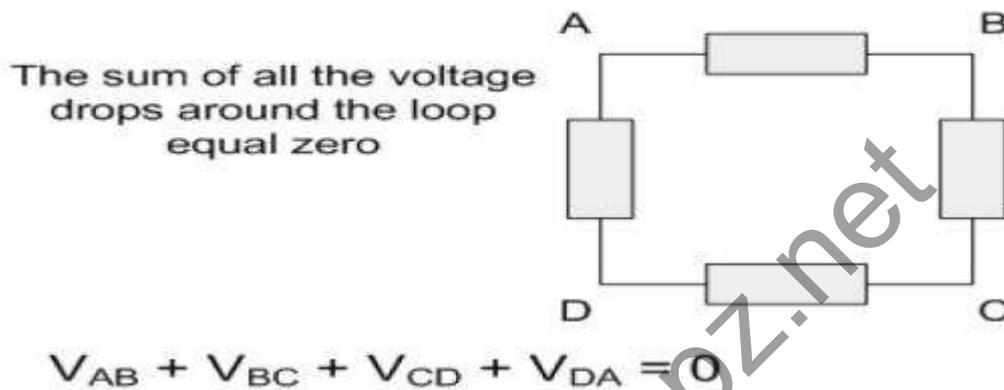
### Kirchoff's Second Law - The Voltage Law, (KVL)

"In any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop" which is also equal to zero.

In other words the algebraic sum of all voltages within the loop must be equal to zero. This idea by Kirchoff is known as the Conservation of Energy.

Starting at any point in the loop continue in the same direction noting the direction of all the voltage drops, either positive or negative, and returning back to the same starting point. It is important to maintain the same direction either clockwise or anti-clockwise or the final voltage sum will not be equal to zero.

We can use Kirchoff's voltage law when analyzing series circuits.



**Problem 1:**

A current of 0.5 A is flowing through the resistance of  $10\Omega$ . Find the potential difference between its ends.

Solution:

Current  $I = 0.5\text{A}$ .

Resistance  $R = 10\Omega$

Potential difference  $V = ?$

$$\begin{aligned} V &= IR \\ &= 0.5 \times 10 \\ &= 5\text{V}. \end{aligned}$$

**Problem: 2**

A supply voltage of 220V is applied to a resistor  $100\Omega$ . Find the current flowing through it.

**Solution:**

$$\text{Voltage } V = 220\text{V Resistance } R = 100\Omega \text{ Current } I = V / R$$

$$= 220 / 100$$

$$= 2.2 \text{ A.}$$

**Problem: 3**

Calculate the resistance of the conductor if a current of 2A flows through it when the potential difference across its ends is 6V.

**Solution:**

$$\text{Current } I = 2\text{A. Potential difference} = V = 6. \text{ Resistance } R = V/I$$

$$= 6 / 2$$

$$= 3 \text{ ohm.}$$

**Problem: 4**

Calculate the current and resistance of a 100 W, 200V electric bulb.

**Solution:**

$$\text{Power, } P = 100\text{W}$$

$$\text{Voltage, } V = 200\text{V Power } p = VI$$

$$\text{Current } I = P/V$$

$$= 100/200$$

$$= 0.5\text{A}$$

$$\text{Resistance } R = V / I$$

$$= 200/0.5$$

$$= 400\text{W.}$$

**Problem: 5**

Calculate the power rating of the heater coil when used on 220V supply *taking 5 Amps.*

**Solution:**

$$\text{Voltage, } V = 220\text{V Current, } I = 5\text{A, Power, } P = VI$$

$$\begin{aligned}
 &= 220 \times 5 \\
 &= 1100\text{W} \\
 &= 1.1 \text{ KW.}
 \end{aligned}$$

**Problem: 6**

A circuit is made of 0.4 wire,  $\Omega$  a 150 bulb  $\Omega$  and a rheostat 120 connected  $\Omega$  in series. Determine the total resistance of the resistance of the circuit.

**Solution:**

Resistance of the wire = 0.4 Resistance  $\Omega$  of bulb = 150  $\Omega$  Resistance of rheostat = 120  $\Omega$

In series,

$$\text{Total resistance, } R = 0.4 + 150 + 120 = 270.4\Omega$$

**Problem : 7**

Three resistances of values 2  $\Omega$ , 3  $\Omega$  connected and in series 5  $\Omega$  across are 20 V ,D.C supply

.Calculate (a) equivalent resistance of the circuit (b) the total current of the circuit (c) the voltage drop across each resistor and (d) the power dissipated in each resistor.

Solution:

$$\begin{aligned}
 \text{Total resistance } R &= R_1 + R_2 + R_3. \\
 &= 2 + 3 + 5 = 10\Omega
 \end{aligned}$$

$$\text{Voltage } = 20\text{V}$$

$$\text{Total current } I = V/R = 20/10 = 2\text{A.}$$

$$\begin{aligned}
 \text{Voltage drop across } 2\Omega \text{ resistor } V_1 &= I R_1 \\
 &= 2 \times 2 = 4 \text{ volts.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Voltage drop across } 3\Omega \text{ resistor } V_2 &= I R_2 \\
 &= 2 \times 3 = 6 \text{ volts.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Voltage drop across } 5\Omega \text{ resistor } V_3 &= I R_3 \\
 &= 2 \times 5 = 10 \text{ volts.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Power dissipated in } 2\Omega \text{ resistor is } P_1 &= I^2 R_1 \\
 &= 2^2 \times 2 = 8 \text{ watts.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Power dissipated in } 3 \text{ resistor is } P_2 &= I^2 R_2. \\
 &= 2^2 \times 3 = 12 \text{ watts.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Power dissipated in } 5 \text{ resistor is } P_3 &= I^2 R_3 \\
 &= 2^2 \times 5 = 20 \text{ watts.}
 \end{aligned}$$

**Problem: 8**

A lamp can work on a 50 volt mains taking 2 amps. What value of the resistance must be connected in series with it so that it can be operated from 200 volt mains giving the same power.

**Solution:**

Lamp voltage,  $V = 50\text{V}$  Current,  $I = 2$  amps.

Resistance of the lamp  $= V/I = 50/2 = 25 \Omega$

Resistance connected in series with lamp  $= r$ .

Supply voltage  $= 200$  volt.

Circuit current  $I = 2\text{A}$

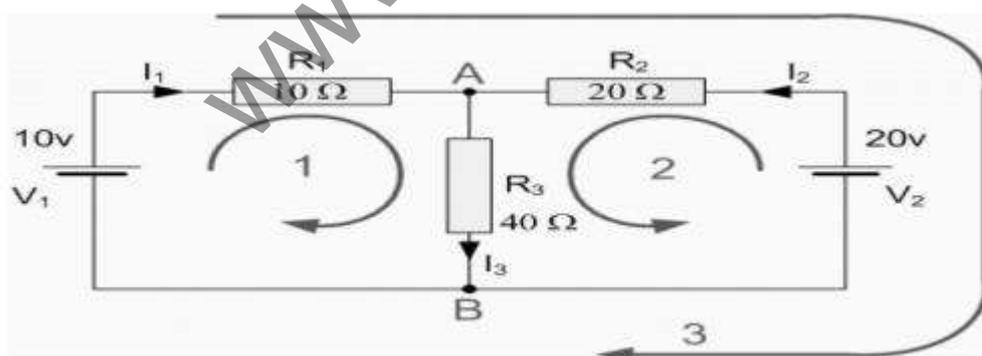
Total resistance  $R_t = V/I = 200/2 = 100\Omega$

$$R_t = R + r \quad 100 = 25 + r$$

$$r = 75\Omega$$

**Problem: 9**

Find the current flowing in the  $40\Omega$  Resistor,



**Solution:**

The circuit has 3 branches, 2 nodes (A and B) and 2 independent loops.

Using Kirchoff's Current Law, KCL the equations are given as;

At node A:  $I_1 + I_2 = I_3$

At node B:  $I_3 = I_1 + I_2$

Using Kirchoff's Voltage Law, KVL the equations are given as;

Loop 1 is given as:  $10 = R_1 \times I_1 + R_3 \times I_3 = 10I_1 + 40I_3$

Loop 2 is given as:  $20 = R_2 \times I_2 + R_3 \times I_3 = 20I_2 + 40I_3$

Loop 3 is given as:  $10 - 20 = 10I_1 - 20I_2$

As  $I_3$  is the sum of  $I_1 + I_2$  we can rewrite the equations as;

Eq. No 1:  $10 = 10I_1 + 40(I_1 + I_2) = 50I_1 + 40I_2$

Eq.No 2:  $20 = 20I_1 + 40(I_1 + I_2) = 40I_1 + 60I_2$

We now have two "Simultaneous Equations" that can be reduced to give us the value of both  $I_1$  and

$I_2$

Substitution of  $I_1$  in terms of  $I_2$  gives us the value of  $I_1$  as  $-0.143$  Amps

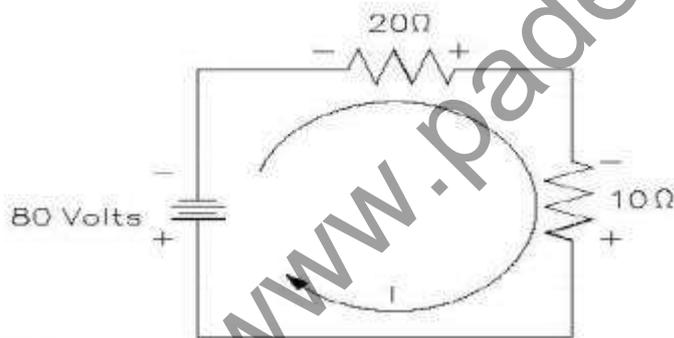
Substitution of  $I_2$  in terms of  $I_1$  gives us the value of  $I_2$  as  $+0.429$  Amps

As:  $I_3 = I_1 + I_2$

The current flowing in resistor  $R_3$  is given as:  $-0.143 + 0.429 = 0.286$  Amps and the voltage across the resistor  $R_3$  is given as :  $0.286 \times 40 = 11.44$  volts

### Problem: 10

Find the current in a circuit using Kirchoff's voltage law



**Solution:**

$$80 = 20(I) + 10(I)$$

$$80 = 30(I)$$

$$I = 80/30 = 2.66 \text{ amperes}$$

### DC CIRCUITS:

A DC circuit (Direct Current circuit) is an electrical circuit that consists of any combination of constant voltage sources, constant current sources, and resistors. In this case, the circuit voltages and currents are constant, i.e., independent of time. More technically, a DC circuit has no memory. That is, a particular circuit voltage or current does not depend on the past value of any circuit voltage or

current. This implies that the system of equations that represent a DC circuit do not involve integrals or derivatives.

If a capacitor and/or inductor is added to a DC circuit, the resulting circuit is not, strictly speaking, a DC circuit. However, most such circuits have a DC solution. This solution gives the circuit voltages and currents when the circuit is in DC steady state. More technically, such a circuit is represented by a system of differential equations. The solutions to these equations usually contain a time varying or transient part as well as constant or steady state part. It is this steady state part that is the DC solution. There are some circuits that do not have a DC solution. Two simple examples are a constant current source connected to a capacitor and a constant voltage source connected to an inductor.

In electronics, it is common to refer to a circuit that is powered by a DC voltage source such as a battery or the output of a DC power supply as a DC circuit even though what is meant is that the circuit is DC powered.

## **AC CIRCUITS:**

### **Fundamentals of AC:**

An alternating current (AC) is an electrical current, where the magnitude of the current varies in a cyclical form, as opposed to direct current, where the polarity of the current stays constant.

The usual waveform of an AC circuit is generally that of a sine wave, as these results in the most efficient transmission of energy. However in certain applications different waveforms are used, such as triangular or square waves.

Used generically, AC refers to the form in which electricity is delivered to businesses and residences. However, audio and radio signals carried on electrical wire are also examples of alternating current. In these applications, an important goal is often the recovery of information encoded (or modulated) onto the AC signal.

## **DIFFERENCE BETWEEN AC AND DC:**

Current that flows continuously in one direction is called direct current. Alternating current (A.C) is the current that flows in one direction for a brief time then reverses and flows in opposite direction for a similar time. The source for alternating current is called a.c generator or alternator.

### **Cycle:**

One complete set of positive and negative values of an alternating quantity is called cycle.

**Frequency:**

The number of cycles made by an alternating quantity per second is called frequency. The unit of frequency is Hertz (Hz)

**Amplitude or Peak value:**

The maximum positive or negative value of an alternating quantity is called amplitude or peak value.

**Average value:**

This is the average of instantaneous values of an alternating quantity over one complete cycle of the wave.

**Time period:**

The time taken to complete one complete cycle.

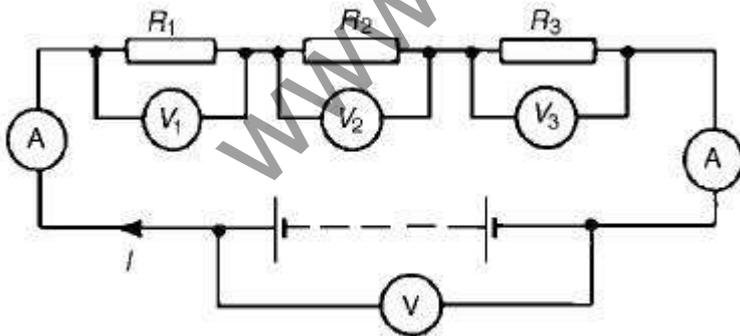
**Average value derivation:**

Let  $i$  = the instantaneous value of current and  $i = I_m \sin \theta$  Where,  $I_m$  is the maximum value.

**Resistors in series and parallel circuits:**

**Series circuits:**

Figure shows three resistors  $R_1$ ,  $R_2$  and  $R_3$  connected end to end, i.e. in series, with a battery source of  $V$  volts. Since the circuit is closed a current  $I$  will flow and the p.d. across each resistor may be determined from the voltmeter readings  $V_1$ ,  $V_2$  and  $V_3$



**In a series circuit**

(a) the current  $I$  is the same in all parts of the circuit and hence the same reading is found on each of the two ammeters shown, and

(b) the sum of the voltages  $V_1$ ,  $V_2$  and  $V_3$  is equal to the total applied voltage,  $V$ , i.e.

$$V = V_1 + V_2 + V_3$$

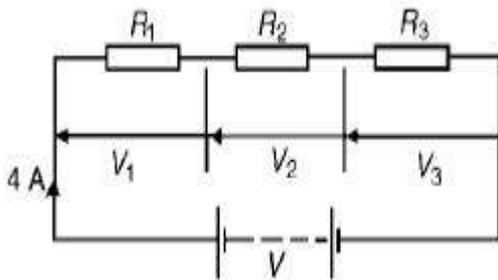
From Ohm's law:

$V_1 = IR_1$ ,  $V_2 = IR_2$ ,  $V_3 = IR_3$  and  $V = IR$  where  $R$  is the total circuit resistance.  
Since  $V = V_1 + V_2 + V_3$

then  $IR = IR_1 + IR_2 + IR_3$  Dividing throughout by  $I$  gives  $R = R_1 + R_2 + R_3$

Thus for a series circuit, the total resistance is obtained by adding together the values of the separate resistances.

**Problem 1:** For the circuit shown in Figure 5.2, determine (a) the battery voltage  $V$ , (b) the total resistance of the circuit, and (c) the values of resistance of resistors  $R_1$ ,  $R_2$  and  $R_3$ , given that the p.d.'s across  $R_1$ ,  $R_2$  and  $R_3$  are  $5V$ ,  $2V$  and  $6V$  respectively.



(a) Battery voltage  $V = V_1 + V_2 + V_3 = 5 + 2 + 6 = 13\text{ V}$

(b) Total circuit resistance  $R = V/I$   
 $= 13/4 = 3.25\ \Omega$

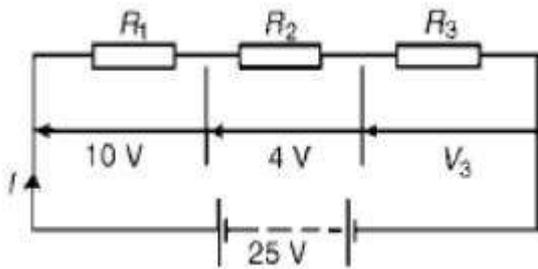
(c) Resistance  $R_1 = V_1/I$   
 $= 5/4$

$= 1.25\ \Omega$  Resistance  $R_2 = V_2/I$

$= 2/4 = 0.5\ \Omega$

Resistance  $R_3 = V_3/I = 6/4 = 1.5\ \Omega$

**Problem 2.** For the circuit shown in Figure determine the p.d. across resistor  $R_3$ . If the total resistance of the circuit is  $100\ \Omega$ , determine the current flowing through resistor  $R_1$ . Find also the value of resistor  $R_2$ .



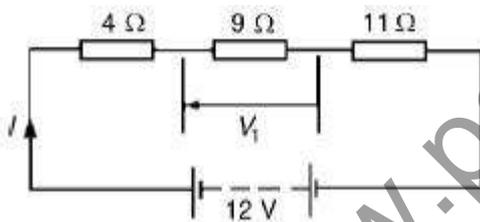
P.d. across  $R_3$ ,  $V_3 = 25 - 10 - 4 = 11V$  Current  $I = V / R$

$$= 25/100$$

$= 0.25A$ , which is the current flowing in each resistor Resistance  $R_2 = V_2 / I$

$$= 4 / 0.25 = 16 \Omega$$

**Problem 3:** A 12V battery is connected in a circuit having three series-connected resistors having resistances of  $4 \Omega$ ,  $9 \Omega$  and  $11 \Omega$ . Determine the current flowing through, and the p.d. across the  $9 \Omega$  resistor. Find also the power dissipated in the  $11 \Omega$  resistor.



Total resistance  $R = 4 + 9 + 11 = 24 \Omega$  Current  $I = V / R$

$$= 12/24$$

$= 0.5A$ , which is the current in the  $9 \Omega$  resistor. P.d. across the  $9 \Omega$  resistor,  $V_1 = I \times 9 = 0.5 \times 9$

$$= 4.5V$$

Power dissipated in the  $11 \Omega$  resistor,  $P = I^2 R = 0.5^2 (11)$

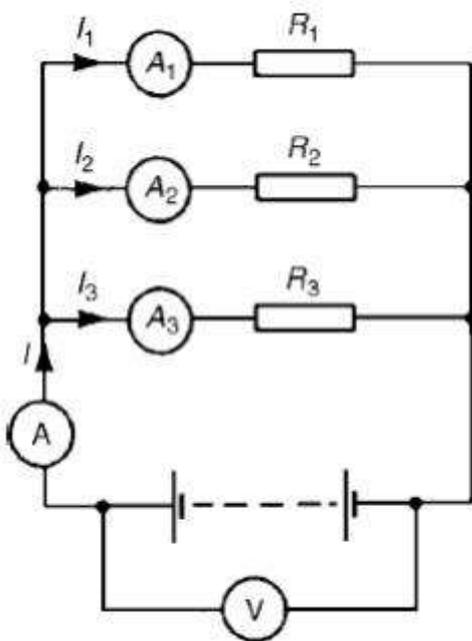
$$= 0.25(11)$$

$$= 2.75W$$

## PARALLEL NETWORKS:

**Problem 1:** Figure shows three resistors,  $R_1$ ,  $R_2$  and  $R_3$  connected across each other, i.e. in parallel, across a battery source

of V volts.



**In a parallel circuit:**

(a) the sum of the currents  $I_1$ ,  $I_2$  and  $I_3$  is equal to the total circuit current,  $I$ , i.e.  $I = I_1 + I_2 + I_3$ , and

the source p.d.,  $V$  volts, is the same across each of the

From Ohm's law:

$$I_1 = V/R_1$$

$$, I_2 = V/R_2$$

$$, I_3 = V/R_3 \text{ and } I = V/R$$

where  $R$  is the total circuit resistance. Since  $I = I_1 + I_2 + I_3$

then

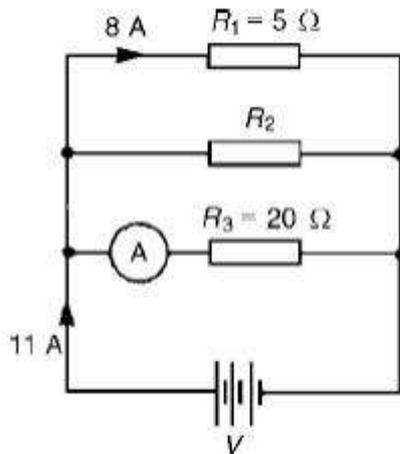
$V/R = V/R_1 + V/R_2 + V/R_3$  Dividing throughout by  $V$  gives:

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

This equation must be used when finding the total resistance  $R$  of a parallel circuit. For the special case of two resistors in parallel

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

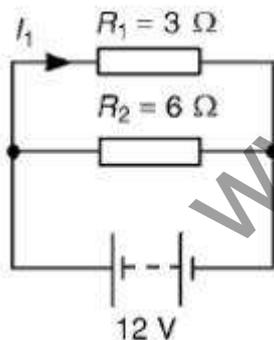
**Problem 2:** For the circuit shown in Figure , determine (a) the reading on the ammeter, and (b) the value of resistor R2.



P.d. across R1 is the same as the supply voltage V.  
Hence supply voltage,  $V = 8 \times 5 = 40\text{V}$

(a) Reading on ammeter,  $I = \frac{V}{R_3} = \frac{40}{20} = 2\text{A}$

Current flowing through R2 =  $11 - 8 - 2 = 1\text{A}$   
Hence,  $R_2 = \frac{V}{I_2} = \frac{40}{1} = 40 \Omega$

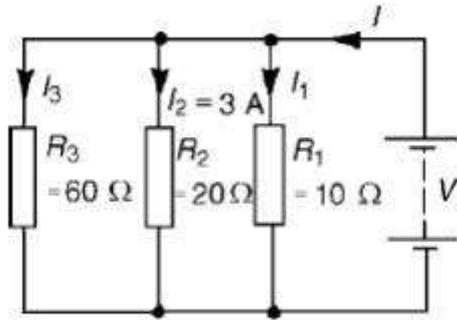


(a) The total circuit resistance R is given by  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{3} + \frac{1}{6}$

$\frac{1}{R} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6}$  Hence,  $R = \frac{6}{3} = 2 \Omega$

(b) Current in the 3 Ω resistance,  $I_1 = \frac{V}{R_1} = \frac{12}{3} = 4\text{A}$

**Problem 3:** For the circuit shown in Figure find (a) the value of the supply voltage V and (b) the value of current I.



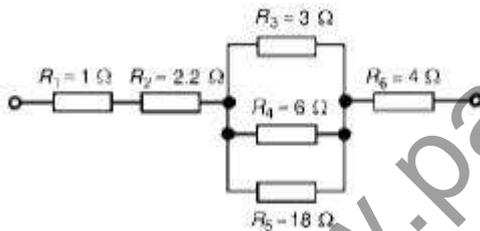
(a) P.d. across  $20\ \Omega$  resistor =  $I_2 R_2 = 3 \times 20 = 60\text{V}$ , hence supply voltage  $V = 60\text{V}$  since the circuit is connected in parallel.

(b) Current  $I_1 = V/R_1 = 60/10 = 6\text{A}$ ;  $I_2 = 3\text{A}$   
 $I_3 = V/R_3 = 60/60 = 1\text{A}$

Current  $I = I_1 + I_2 + I_3$  and hence  $I = 6 + 3 + 1 = 10\text{A}$  Alternatively,

$1/R = 1/60 + 1/20 + 1/10 = 1 + 3 + 6/60 = 10/60$  Hence total resistance  $R = 60/10 = 6\ \Omega$   
 Current  $I = V/R = 60/6 = 10\text{A}$

**Problem 4:** Find the equivalent resistance for the circuit shown in Figure



$R_3$ ,  $R_4$  and  $R_5$  are connected in parallel and their equivalent resistance  $R$  is given by:  $1/R = 1/3 + 1/6 + 1/18 = 6 + 3 + 1/18 = 10/18$

Hence  $R = 18/10 = 1.8\ \Omega$

The circuit is now equivalent to four resistors in series and the equivalent circuit resistance =  $1 + 2.2 + 1.8 + 4 = 9\ \Omega$

### MESH ANALYSIS:

This is an alternative structured approach to solving the circuit and is based on calculating mesh currents. A similar approach to the node situation is used. A set of equations (based on KVL for each mesh) is formed and the equations are solved for unknown values. As many equations are needed as unknown mesh currents exist.

Step 1: Identify the mesh currents

Step 2: Determine which mesh currents are known

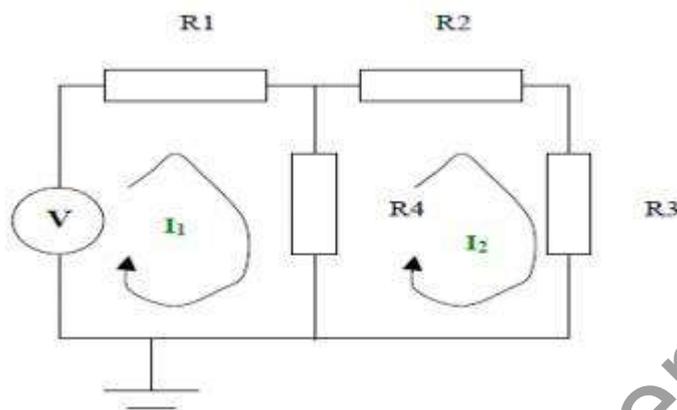
Step 2: Write equation for each mesh using KVL and that includes the mesh currents  
 Step 3: Solve the equations

**Step 1:**

The mesh currents are as shown in the diagram on the next page

**Step 2:**

Neither of the mesh currents is known



**Step 3:**

KVL can be applied to the left hand side loop. This states the voltages around the loop sum to zero.

When writing down the voltages across each resistor equations are the mesh currents.

$$I_1 R_1 + (I_1 - I_2) R_4 - V = 0$$

KVL can be applied to the right hand side loop. This states the voltages around the loop sum to

zero. When writing down the voltages across ea the equations are the mesh currents.

$$I_2 R_2 + I_2 R_3 + (I_2 - I_1) R_4 = 0$$

**Step 4:**

Solving the equations we get

$$I_1 = V \frac{R_2 + R_3 + R_4}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_4 + R_3 R_4}$$

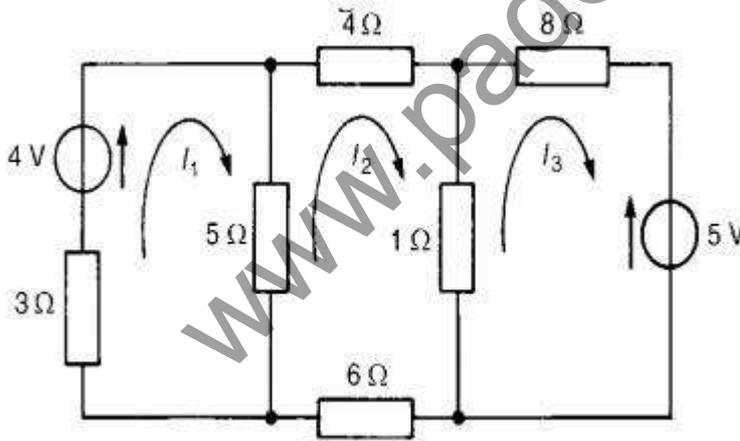
$$I_2 = V \frac{R_4}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_4 + R_3 R_4}$$

The individual branch currents can be obtained from these mesh currents and the node voltages can also be calculated using this information. For example:

$$V_C = I_2 R_3 = V \frac{R_3 R_4}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_4 + R_3 R_4}$$

**Problem 1:**

Use mesh-current analysis to determine the current flowing in (a) the 1Ω resistance of the d.c. circuit shown in



The mesh currents  $I_1$ ,  $I_2$  and  $I_3$  are shown in Figure

Using Kirchhoff's voltage law:

For loop 1,  $(3 + 5) I_1 - I_2 = 4$  .....(1)

For loop 2,  $(4 + 1 + 6 + 5) I_2 - (5) I_1 - (1) I_3 = 0$  .....(2)

For loop 3,  $(1 + 8) I_3 - (1) I_2 = 5$  .....(3)

Thus

$$8I_1 - 5I_2 - 4 = 0$$

$$-5I_1 + 16I_2 - I_3 = 0$$

$$-I_2 + 9I_3 + 5 = 0$$

$$\frac{I_1}{\begin{vmatrix} -5 & 0 & -4 \\ 16 & -1 & 0 \\ -1 & 9 & 5 \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} 8 & 0 & -4 \\ -5 & -1 & 0 \\ 0 & 9 & 5 \end{vmatrix}} = \frac{I_3}{\begin{vmatrix} 8 & -5 & -4 \\ -5 & 16 & 0 \\ 0 & -1 & 5 \end{vmatrix}}$$

$$= \frac{-1}{\begin{vmatrix} 8 & -5 & 0 \\ -5 & 16 & -1 \\ 0 & -1 & 9 \end{vmatrix}}$$

Using determinants,

$$\frac{I_1}{-5 \begin{vmatrix} -1 & 0 \\ 9 & 5 \end{vmatrix} - 4 \begin{vmatrix} 16 & -1 \\ -1 & 9 \end{vmatrix}} = \frac{-I_2}{8 \begin{vmatrix} -1 & 0 \\ 9 & 5 \end{vmatrix} - 4 \begin{vmatrix} -5 & -1 \\ 0 & 9 \end{vmatrix}}$$

$$= \frac{I_3}{-4 \begin{vmatrix} -5 & 16 \\ 0 & -1 \end{vmatrix} + 5 \begin{vmatrix} 8 & -5 \\ -5 & 16 \end{vmatrix}}$$

$$= \frac{-1}{8 \begin{vmatrix} 16 & -1 \\ -1 & 9 \end{vmatrix} + 5 \begin{vmatrix} -5 & -1 \\ 0 & 9 \end{vmatrix}}$$

$$\frac{I_1}{-5(-5) - 4(143)} = \frac{-I_2}{8(-5) - 4(-45)}$$

$$= \frac{I_3}{-4(5) + 5(103)}$$

$$\begin{aligned} &= \frac{I_3}{-4(5) + 5(103)} \\ &= \frac{-1}{8(143) + 5(-45)} \\ \frac{I_1}{-547} &= \frac{-I_2}{140} = \frac{I_3}{495} = \frac{-1}{919} \end{aligned}$$

Hence  $I_1 = \frac{547}{919} = 0.595 \text{ A}$ ,

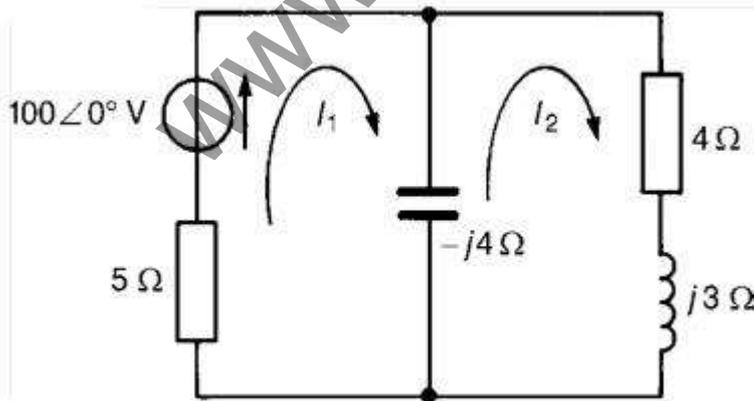
$I_2 = \frac{140}{919} = 0.152 \text{ A}$ , and

$I_3 = \frac{-495}{919} = -0.539 \text{ A}$

(a) Current in the  $5 \Omega$  resistance =  $I_1 - I_2$   
 $= 0.595 - 0.152$   
 $= 0.44 \text{ A}$

(b) Current in the  $1 \Omega$  resistance =  $I_2 - I_3$   
 $= 0.152 - (-0.539)$   
 $= 0.69 \text{ A}$

**Problem 2:** For the a.c. network shown in Figure determine, using mesh-current analysis, (a) the mesh currents  $I_1$  and  $I_2$  (b) the current flowing in the capacitor, and (c) the active power delivered by the  $100 \angle 0^\circ \text{ V}$  voltage source.



(a) For the first loop  
 $(5 - j4) I_1 - (-j4) I_2 = 100 \angle 0^\circ$ .....(1) For the second loop

$$(4+j3-j4)I_2 = 0 \dots\dots\dots (2) \quad -(-j4I_1)$$

Rewriting equations (1) and (2) gives:

$$(5 -j4)I_1 + j4I_2 -100 =0$$

$j4I_1 + (4 -j) I_2 + 0 =0$  Thus, using determinants,

(b) Current flowing

$$\begin{aligned} &= I_1 - I_2 \\ &= 10.77 \angle 19. \\ &= 4.44 + j12 \end{aligned}$$

$$\frac{I_1}{\begin{vmatrix} j4 & -100 \\ (4-j) & 0 \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} (5-j4) & -100 \\ j4 & 0 \end{vmatrix}}$$

$$= \frac{1}{\begin{vmatrix} (5-j4) & j4 \\ j4 & (4-j) \end{vmatrix}}$$

i.e. the current

(c) Source power  $P$

$$\frac{I_1}{(400 -j100)} = \frac{-I_2}{j400} = \frac{1}{(32 -j21)}$$

$$\begin{aligned} \text{Hence } I_1 &= \frac{(400 -j100)}{(32 -j21)} = \frac{412.31 \angle -14.04^\circ}{38.28 \angle -33.27^\circ} \\ &= 10.77 \angle 19.23^\circ \text{ A} = 10.8 \angle -19.2^\circ \text{ A,} \end{aligned}$$

(Check: power i

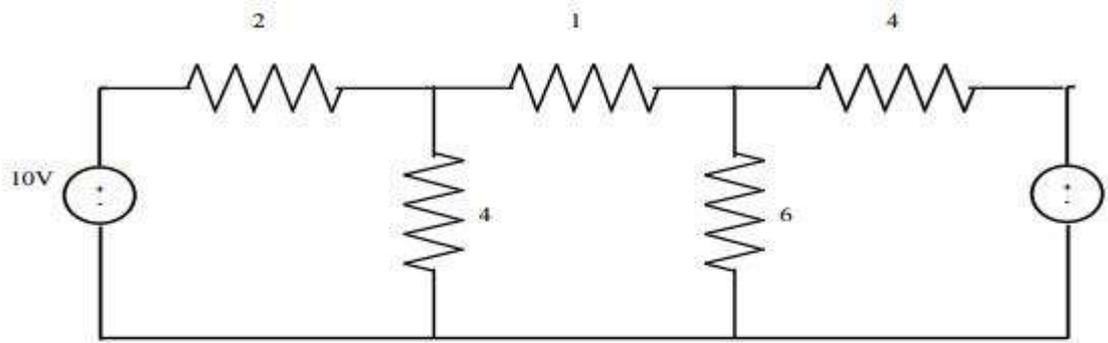
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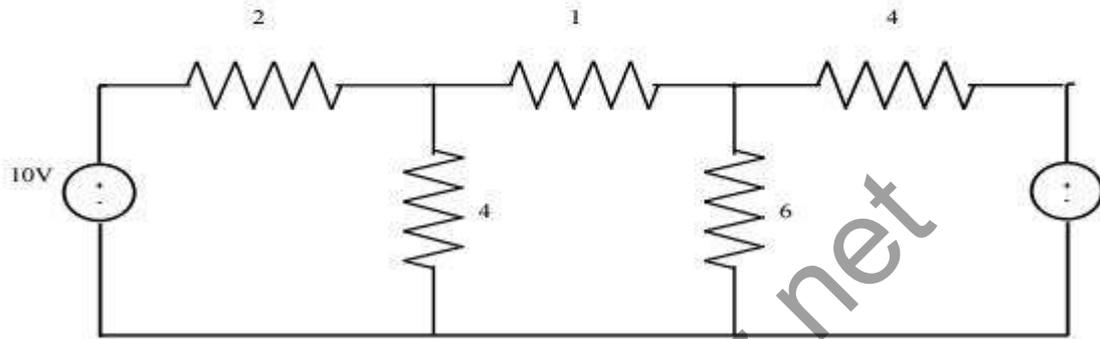
$$\begin{aligned} I_2 &= \frac{400 \angle -90^\circ}{38.28 \angle -33.27^\circ} = 10.45 \angle -56.73^\circ \text{ A} \\ &= 10.5 \angle -56.7^\circ \text{ A,} \end{aligned}$$

Thus total power dissipated = 579.97 + 436.81 = 1016.8W = 1020W

**Problem 3:** Calculate current through  $-6\Omega$  resistance u



**Solution:**



Case(1): Consider loop ABGH ; Apply KVL .

Case(1): Consider loop ABGH ; Apply KVL .

$$10 = 2I_1 + 4(I_1 - I_2)$$

$$10 = 6I_1 - 4I_2 \text{ ----- (1)}$$

Consider loop BCFG

$$I_2 + 6(I_2 + I_3) + 4(I_2 - I_1) = 0$$

$$11I_2 + 6I_3 - 4I_1 = 0 \text{ ----- (2)}$$

Consider loop CDEF

$$20 = 4I_3 + 6(I_2 - I_3)$$

$$20 = 10I_3 + 6I_2 \text{ ----- (3)}$$

$$D = \begin{vmatrix} 6 & -4 & 0 \\ -4 & 11 & 6 \\ 0 & 6 & 10 \end{vmatrix}$$

$$= \begin{vmatrix} 10 \\ 0 \\ 20 \end{vmatrix}$$

$$D = [6(110 - 36) + 4(-40)] = 284.$$

$$D_1 = \begin{vmatrix} 10 & -4 & 0 \\ 0 & 11 & 6 \\ 20 & 6 & 10 \end{vmatrix}$$

$$D_1 = 10[110 - 36 + (-120)]$$

$$= 260$$

$$D_2 = \begin{vmatrix} 6 & 10 & 0 \\ -4 & 0 & 6 \\ 0 & 20 & 10 \end{vmatrix}$$

$$D_2 = 6(-120) - 10(-40) = -320$$

$$D_3 = \begin{vmatrix} 6 & -4 & 10 \\ -4 & 11 & 0 \\ 0 & 6 & 20 \end{vmatrix}$$

$$D_3 = 6(220) + 4(-80) + 10(-24)$$

$$D_3 = 760$$

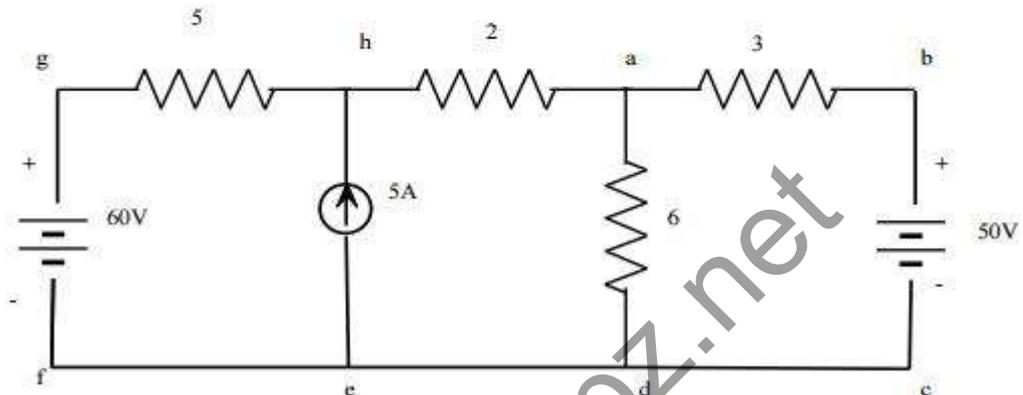
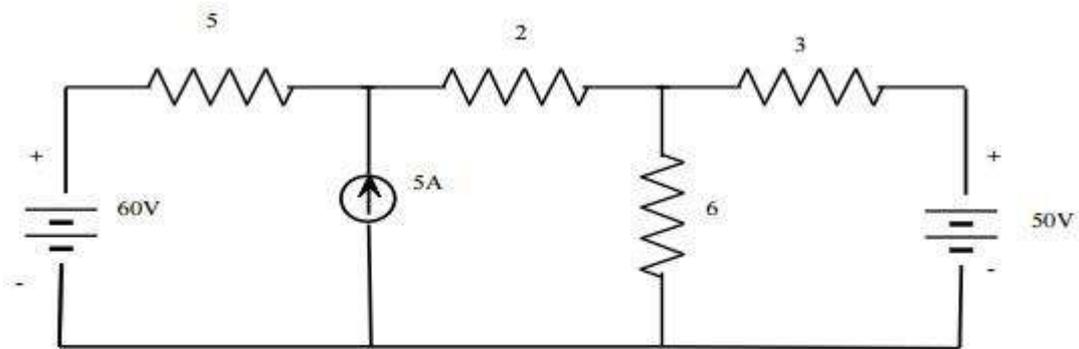
$$I_1 = D_1/D = 260/284 = 0.915A$$

$$I_2 = D_2/D = -320/284 = -1.1267A \quad I_3 = D_3/D = 760/284 = 2.676A$$

$$\text{Current through } 6\Omega \text{ 2 +resistance3} = I$$

$$= -1.1267 + 2.676 = 1.55A$$

**Problem 4:** Find the current through branch a-b using mesh analysis.



Solution:

Consider loops

Loop HADE  $\rightarrow 5I_1 + 2I_2 + 6(I_2 - I_3) = 60$

$$5I_1 + 8I_2 - 6I_3 = 60 \text{ ----- (1)}$$

Loop ABCDA  $\rightarrow 3I_3 + 6(I_3 - I_2) = -50$

$$3I_3 + 6I_3 - 6I_2 = -50$$

$$9I_3 - 6I_2 = -50 \text{ ----- (2)}$$

$$I_2 - I_1 = 5A \text{ ----- (3)}$$

From (1), (2) & (3).

$$D = \begin{vmatrix} -1 & 1 & 0 \\ 5 & 8 & -6 \\ 0 & -6 & 9 \end{vmatrix}$$

$$= -1(72-36) - 1(45)$$

$$D = -81$$

$$D_3 = \begin{vmatrix} -1 & 1 & 5 \\ 5 & 8 & 60 \\ 0 & -6 & -50 \end{vmatrix}$$

$$= -1(-400+360) - (-250) + 5(-30)$$

$$\begin{aligned} &= 40+250-150 \\ D3 &= 140. \\ I3 &= D3/D = 140/-81 = -1.7283 \end{aligned}$$

The current through branch ab is 1.7283A which is flowing from b to a.

### **NODAL ANALYSIS:**

Nodal analysis involves looking at a circuit and determining all the node voltages in the circuit. The voltage at any given node of a circuit is the voltage drop between that node and a reference node (usually ground). Once the node voltages are known any of the currents flowing in the circuit can be determined. The node method offers an organized way of achieving this.

### **Approach:**

Firstly all the nodes in the circuit are counted and identified. Secondly nodes at which the voltage is already known are listed. A set of equations based on the node voltages are formed and these equations are solved for unknown quantities. The set of equations are formed using KCL at each node. The set of simultaneous equations that is produced is then solved. Branch currents can then be found once the node voltages are known. This can be reduced to a series of steps:

Step 1: Identify the nodes

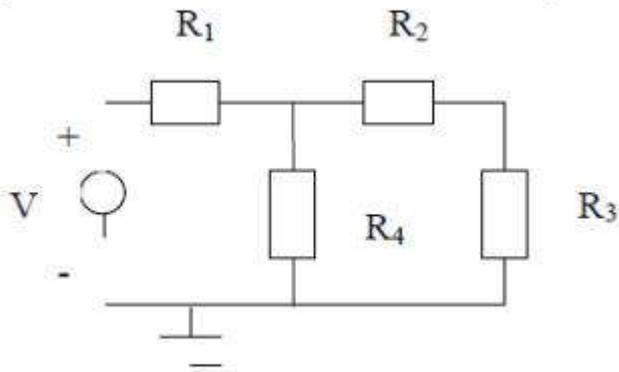
Step 2: Choose a reference node

Step 3: Identify which node voltages are known if any Step 4: Identify the BRANCH currents

Step 5: Use KCL to write an equation for each unknown node voltage Step

6: Solve the equations

This is best illustrated with an example. Find all currents and voltages in the following circuit using the node method. (In this particular case it can be solved in other ways as well)



**Step 1:**

There are four nodes in the circuit. A, B, C and D

**Step 2:**

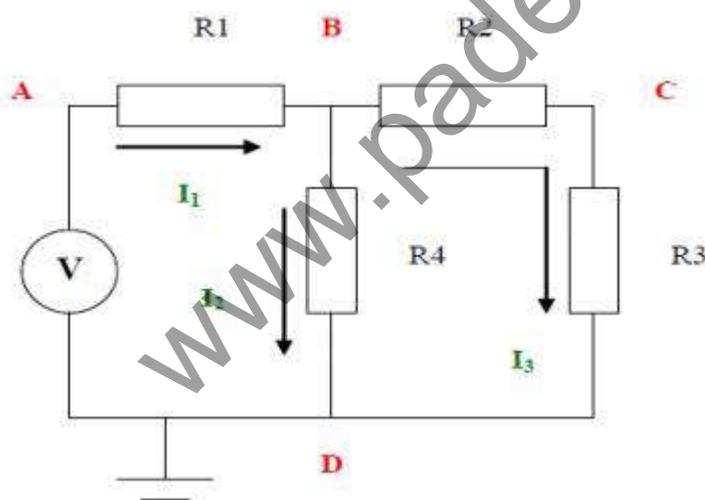
Ground, node D is the reference node.

**Step 3:**

Node voltage B and C are unknown. Voltage at A is V and at D is 0

**Step 4:**

The currents are as shown. There are 3 different currents



**Step 5:**

I need to create two equations so I apply KCL at node B and node C

The statement of KCL for node B is as follows:

$$\frac{V - V_B}{R_1} + \frac{V_C - V_B}{R_2} + \frac{-V_B}{R_4} = 0$$

The statement of KCL for node C is as follows:

$$\frac{V_C - V_B}{R_2} + \frac{-V_B}{R_3} = 0$$

**Step 6:**

We now have two equations to solve for the two unknowns  $V_B$  and  $V_C$ . Solving the above two equations we get:

$$V_C = V \frac{R_3 R_4}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_4 + R_3 R_4}$$

$$V_B = V \frac{R_4 (R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_4 + R_3 R_4}$$

**Further Calculations**

The node voltages are now all known. From these we can get the branch currents by a simple application of Ohm's Law:

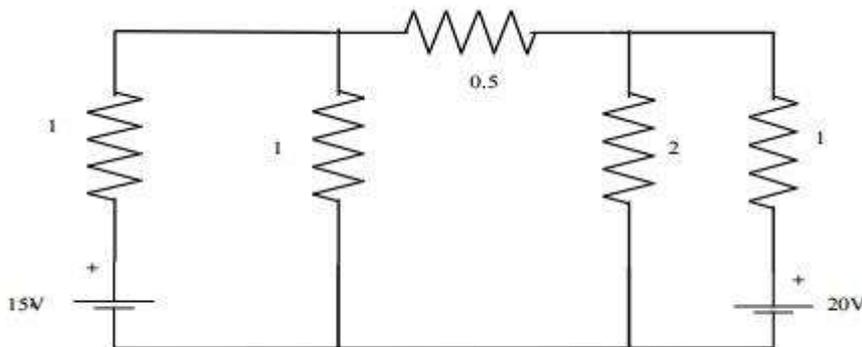
$$I_1 = (V - V_B) / R_1$$

$$I_2 = (V_B - V_C) / R_2$$

$$I_3 = (V_C) / R_3$$

$$I_4 = (V_B) / R_4$$

**Problem 1:** Find the current through each resistor of the circuit shown in fig, using nodal analysis



**Solution:**

**At node1,**

$$-I_1 - I_2 - I_3 = 0 \quad -[V_1 - 15/1] - [V_1/1] - [V_1 - V_2/0.5] = 0$$

$$-V_1 + 15 - V_1 - 2V_1 + 2V_2 = 0$$

$$4V_1 - 2V_2 = 15 \quad \text{----- (1)}$$

**At node2,**

$$I_3 - I_4 - I_5 = 0$$

$$V_1 - V_2/0.5 - V_2/2 - V_2 - 20/1 = 0$$

$$2V_1 - 2V_2 - 0.5V_2 - V_2 + 20 = 0$$

$$2V_1 - 3.5V_2 = -20 \quad \text{----- (2)}$$

Multiplying (2) by 2 & subtracting from (1)

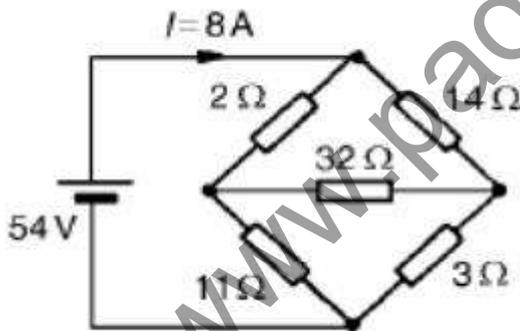
$$5V_2 = 55 \quad V_2 = 11V \quad V = 9.25V$$

$$I_1 = V_1 - 15/1 = 9.25 - 15 = -5.75A = 5.75A \quad I_2 = V_1/1 = 9.25A$$

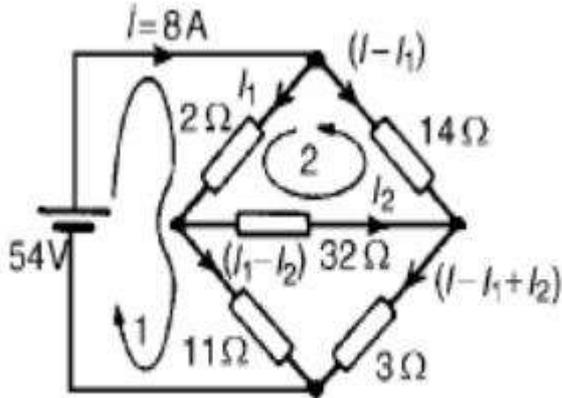
$$I_3 = V_1 - V_2/0.5 = -3.5A = 3.5A \quad \leftarrow I_4 = V_2/2 = 5.5A$$

$$I_5 = V_2 - 20/1 = 11 - 20/1 = -9A = 9A.$$

**Problem 2:** For the bridge network shown in Figure determine the currents in each of the resistors.



Let the current in the  $2\Omega$  resistor be  $I_1$ , and then the current by Kirchhoff's current law in the  $14\Omega$  resistor is  $(I - I_1)$ . Let the current in the  $32\Omega$  resistor be  $I_2$  as shown in Figure. Then the current in the  $1\Omega$  resistor is  $(I_1 - I_2)$  and that in the  $3\Omega$  resistor is  $(I - I_1 + I_2)$ . Applying Kirchhoff's and moving in a clockwise direction as shown in Figure gives:



$$54 = 2I_1 + 11(I_1 - I_2)$$

i.e.  $13I_1 - 11I_2 = 54$

Applying Kirchhoff's voltage law to loop 2 and direction as shown in Figure gives:

$$0 = 2I_1 + 32I_2 - 14(I - I_1)$$

However  $I = 8 \text{ A}$

Hence  $0 = 2I_1 + 32I_2 - 14(8 - I_1)$  i.e.  $16I_1 + 32I_2 = 112$

Equations (1) and (2) are simultaneous equations with two unknowns,  $I_1$  and  $I_2$ .

$16 * (1)$  gives:  $208I_1 - 176I_2 = 864$

$16 * (2)$  gives:  $208I_1 - 176I_2 = 864$

$13 * (2)$  gives:  $208I_1 + 416I_2 = 1456$

$(4) - (3)$  gives:  $592I_2 = 592, I_2 = 1 \text{ A}$

Substituting for  $I_2$  in (1) gives:

$$13I_1 - 11 = 54$$

$$I_1 = 65/13 = 5 \text{ A}$$

Hence,

the current flowing in the 2 Ω resistor =  $I_1 = 5 \text{ A}$

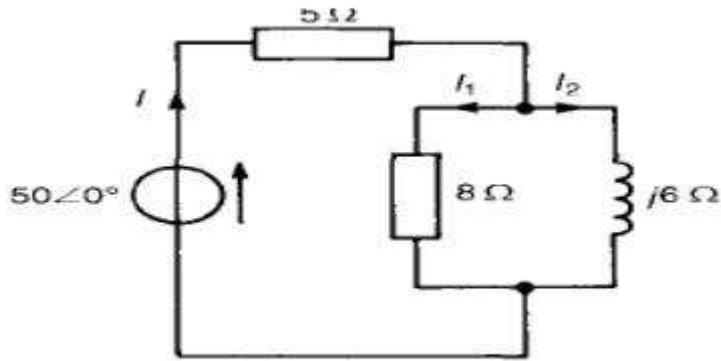
the current flowing in the 14 Ω resistor =  $I - I_1 = 8 - 5 = 3 \text{ A}$

the current flowing in the 32 Ω resistor =  $I_2 = 1 \text{ A}$

the current flowing in the 11 Ω resistor =  $I_1 - I_2 = 5 - 1 = 4 \text{ A}$  and

the current flowing in the 3 Ω resistor =  $I - I_1 + I_2 = 8 - 5 + 1 = 4 \text{ A}$

Problem 3: Determine the values of currents  $I$ ,  $I_1$  and  $I_2$  shown in the network of Figure



Total circuit impedance,

$$\begin{aligned} Z_T &= 5 + (8)(j6)/8 + j6 \\ &= 5 + (j48)(8 - j6)/82 + 62 \\ &= 5 + (j384 + 288)/100 \end{aligned}$$

$$= (7.88 + j3.84) \text{ or } 8.776 \angle 25.98^\circ \text{ A Current } I = V/Z_T$$

$$\begin{aligned} &= \frac{50 \angle 0^\circ}{8.77 \angle 25.98^\circ} \\ &= 5.7066 \angle -25.98^\circ \text{ A} \end{aligned}$$

Current  $I_1 = I (j6/8 + j6)$

$$\begin{aligned} &= \frac{(5.702 \angle -5.98^\circ) (6 \angle 90^\circ)}{10 \angle 36.87^\circ} \\ &= 3.426 \angle 27.15^\circ \text{ A} \end{aligned}$$

Current  $I_2 = I (8 / (8 + j6))$

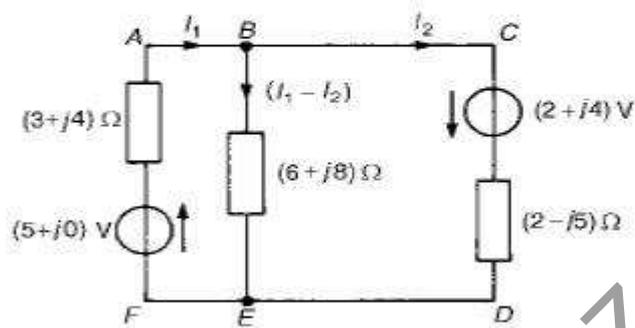
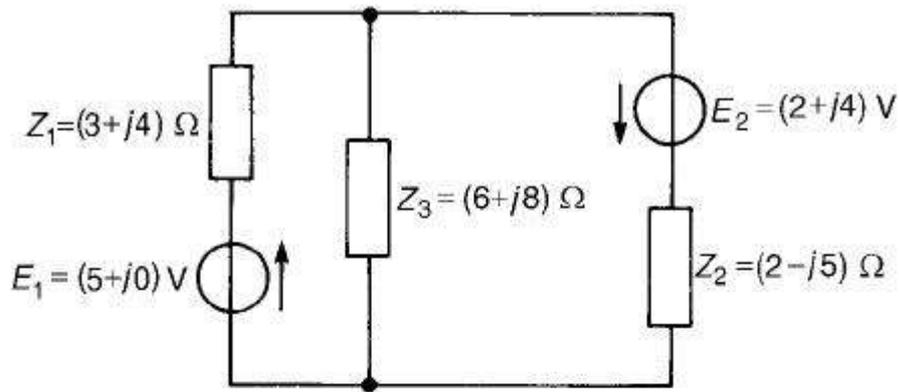
$$\begin{aligned} &= \frac{(5.2570 \angle -9.8^\circ) * 8 \angle 0^\circ}{10 \angle 36.87^\circ} \\ &= 4.5666 \angle -62.85^\circ \text{ A} \end{aligned}$$

[Note:  $I = I_1 + I_2 = 3.42 \angle 27.15^\circ + 4.56 \angle -62.85^\circ$

$$= 3.043 + j1.561 + 2.081 - j4.058$$

$$= 5.124 - j2.497 \text{ A} = 5.706 \angle -25.98^\circ \text{ A}$$

**Problem 4:** For the a.c. network shown in Figure, determine the current flowing in each branch using Kirchhoff's laws.



from which,  $I_1 = \frac{20 + j55}{64 + j27} = \frac{58.52 \angle 70.02^\circ}{69.46 \angle 22.87^\circ} = 0.842 \angle 47.15^\circ \text{ A}$   
 $= (0.573 + j0.617) \text{ A}$   
 $= (0.57 + j0.62) \text{ A, correct to two decimal places.}$

From equation (1),  $5 = (9 + j12)(0.573 + j0.617) - (6 + j8)I_2$

$$5 = (-2.247 + j12.429) - (6 + j8)I_2$$

from which,  $I_2 = \frac{-2.247 + j12.429 - 5}{6 + j8}$

$$= \frac{14.39 \angle 120.25^\circ}{10 \angle 53.13^\circ}$$

$$= 1.439 \angle 67.12^\circ \text{ A} = (0.559 + j1.326) \text{ A}$$

$$= (0.56 + j1.33) \text{ A, correct to two decimal places.}$$

The current in the  $(6 + j8)\Omega$  impedance,

$$I_1 - I_2 = (0.573 + j0.617) - (0.559 + j1.326)$$

$$= (0.014 - j0.709) \text{ A or } 0.709 \angle -88.87^\circ \text{ A}$$

An alternative method of solving equations (1) and (2) is shown below, using determinants.

$$(9 + j12)I_1 - (6 + j8)I_2 - 5 = 0 \tag{1}$$

$$-(6 + j8)I_1 + (8 + j3)I_2 - (2 + j4) = 0 \tag{2}$$

$$\text{Thus } \frac{I_1}{\begin{vmatrix} -(6+j8) & -5 \\ (8+j3) & -(2+j4) \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} (9+j12) & -5 \\ -(6+j8) & -(2+j4) \end{vmatrix}}$$

$$= \frac{1}{\begin{vmatrix} (9+j12) & -(6+j8) \\ -(6+j8) & (8+j3) \end{vmatrix}}$$

$$\frac{I_1}{(-20+j40) + (40+j15)} = \frac{-I_2}{(30-j60) - (30+j40)}$$

$$= \frac{1}{(36+j123) - (-28+j96)}$$

$$\frac{I_1}{20+j55} = \frac{-I_2}{-j100} = \frac{1}{64+j27}$$

$$\text{Hence } I_1 = \frac{20+j55}{64+j27} = \frac{58.52\angle 70.02^\circ}{69.46\angle 22.87^\circ}$$

$$= 0.842\angle 47.15^\circ \text{ A}$$

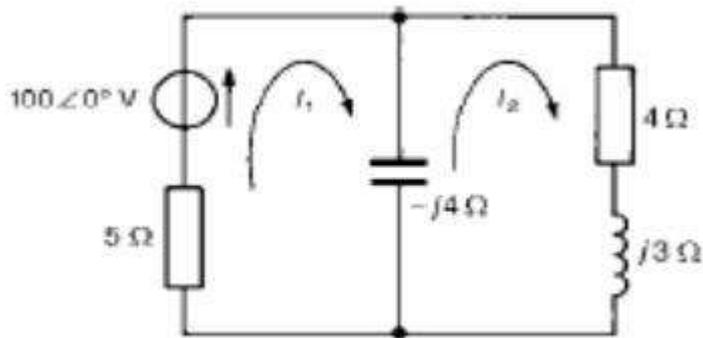
$$\text{and } I_2 = \frac{100\angle 90^\circ}{69.46\angle 22.87^\circ} = 1.440\angle 67.13^\circ \text{ A}$$

The current flowing in the  $(6+j8) \Omega$  impedance is given by:

$$I_1 - I_2 = 0.842\angle 47.15^\circ - 1.440\angle 67.13^\circ \text{ A}$$

$$= (0.013 - j0.709) \text{ A or } 0.709\angle -88.95^\circ \text{ A}$$

**Problem 5:** For the a.c. network shown in Figure determine, using mesh-current analysis, (a) the mesh currents  $I_1$  and  $I_2$  (b) the current flowing in the capacitor, and (c) the active power delivered by the  $100\angle 0^\circ$  V voltage source.



(a) For the first loop  $(5 - j4)I_1 - (-j4I_2) = 100\angle 0^\circ$  (1)

For the second loop  $(4 + j3 - j4)I_2 - (-j4I_1) = 0$  (2)

Rewriting equations (1) and (2) gives:

$$(5 - j4)I_1 + j4I_2 - 100 = 0 \quad (1')$$

$$j4I_1 + (4 - j)I_2 + 0 = 0 \quad (2')$$

Thus, using determinants,

$$\frac{I_1}{\begin{vmatrix} j4 & -100 \\ (4 - j) & 0 \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} (5 - j4) & -100 \\ j4 & 0 \end{vmatrix}} = \frac{1}{\begin{vmatrix} (5 - j4) & j4 \\ j4 & (4 - j) \end{vmatrix}}$$

$$\frac{I_1}{(400 - j100)} = \frac{-I_2}{j400} = \frac{1}{(32 - j21)}$$

Hence  $I_1 = \frac{(400 - j100)}{(32 - j21)} = \frac{412.31\angle -14.04^\circ}{38.28\angle -33.27^\circ}$

$$= 10.77\angle 19.23^\circ \text{ A} = 10.8\angle -19.2^\circ \text{ A,}$$

correct to one decimal place

$$I_2 = \frac{400\angle -90^\circ}{38.28\angle -33.27^\circ} = 10.45\angle -56.73^\circ \text{ A}$$

$$= 10.5\angle -56.7^\circ \text{ A,}$$

correct to one decimal place

(b) Current flowing in capacitor  $= I_1 - I_2$

$$= 10.77 \angle 19.23^\circ - 10.45 \angle -56.73^\circ$$

$$= 4.44 + j12.28 = 13.1 \angle 70.12^\circ \text{ A,}$$

i.e., the current in the capacitor is 13.1 A

(c) Source power  $P = VI \cos \phi = (100)(10.77) \cos 19.23^\circ$

$$= 1016.9 \text{ W} = 1020 \text{ W,}$$

correct to three significant figures.

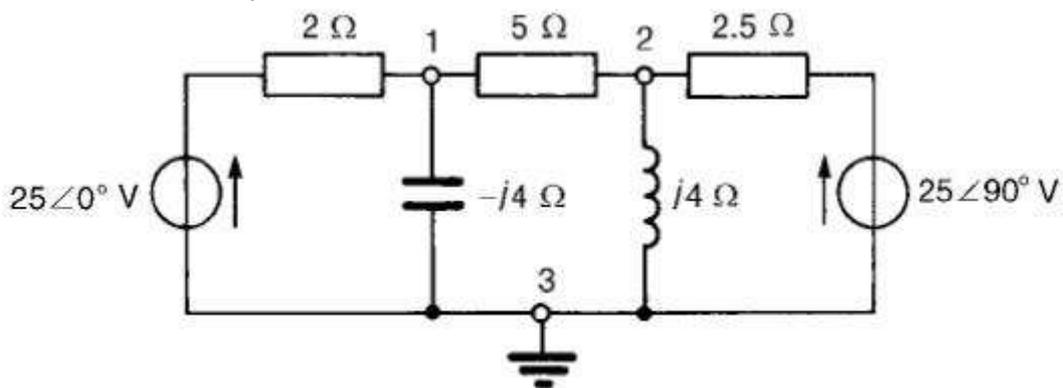
(Check: power in  $5 \Omega$  resistor  $= I_1^2(5) = (10.77)^2(5) = 579.97 \text{ W}$

and power in  $4 \Omega$  resistor  $= I_2^2(4) = (10.45)^2(4) = 436.81 \text{ W}$

Thus total power dissipated  $= 579.97 + 436.81$

$= 1016.8 \text{ W} = 1020 \text{ W, correct to three significant figures.})$

**Problem 6:** In the network of Figure use nodal analysis to determine (a) the voltage at nodes 1 and 2, (b) the current in the  $j4 \Omega$  inductance, (c) of the active power dissipated-10) in the  $2.5 \Omega$



(a) At node 1,  $\frac{V_1 - 25\angle 0^\circ}{2} + \frac{V_1}{-j4} + \frac{V_1 - V_2}{5} = 0$

Rearranging gives:

$$\left(\frac{1}{2} + \frac{1}{-j4} + \frac{1}{5}\right)V_1 - \left(\frac{1}{5}\right)V_2 - \frac{25\angle 0^\circ}{2} = 0$$

i.e.,  $(0.7 + j0.25)V_1 - 0.2V_2 - 12.5 = 0$  (1)

At node 2,  $\frac{V_2 - 25\angle 90^\circ}{2.5} + \frac{V_2}{j4} + \frac{V_2 - V_1}{5} = 0$

Rearranging gives:

$$-\left(\frac{1}{5}\right)V_1 + \left(\frac{1}{2.5} + \frac{1}{j4} + \frac{1}{5}\right)V_2 - \frac{25\angle 90^\circ}{2.5} = 0$$

i.e.,  $-0.2V_1 + (0.6 - j0.25)V_2 - j10 = 0$  (2)

Thus two simultaneous equations have been formed with two unknowns,  $V_1$  and  $V_2$ . Using determinants, if

$$(0.7 + j0.25)V_1 - 0.2V_2 - 12.5 = 0$$
 (1)

and  $-0.2V_1 + (0.6 - j0.25)V_2 - j10 = 0$  (2)

then

$$\begin{aligned} \frac{V_1}{\begin{vmatrix} -0.2 & -12.5 \\ (0.6 - j0.25) & -j10 \end{vmatrix}} &= \frac{-V_2}{\begin{vmatrix} (0.7 + j0.25) & -12.5 \\ -0.2 & -j10 \end{vmatrix}} \\ &= \frac{1}{\begin{vmatrix} (0.7 + j0.25) & -0.2 \\ -0.2 & (0.6 - j0.25) \end{vmatrix}} \end{aligned}$$

i.e.,

$$\frac{V_1}{(j2 + 7.5 - j3.125)} = \frac{-V_2}{(-j7 + 2.5 - 2.5)}$$

$$= \frac{1}{(0.42 - j0.175 + j0.15 + 0.0625 - 0.04)}$$

and  $\frac{V_1}{7.584\angle-8.53^\circ} = \frac{-V_2}{-7\angle90^\circ} = \frac{1}{0.443\angle-3.23^\circ}$

Thus voltage,  $V_1 = \frac{7.584\angle-8.53^\circ}{0.443\angle-3.23^\circ} = 17.12\angle-5.30^\circ \text{ V}$

$= 17.1\angle-5.3^\circ \text{ V}$ , correct to one decimal place,

and voltage,  $V_2 = \frac{7\angle90^\circ}{0.443\angle-3.23^\circ} = 15.80\angle93.23^\circ \text{ V}$

$= 15.8\angle93.2^\circ \text{ V}$ , correct to one decimal place.

(b) The current in the  $j4 \Omega$  inductance is given by:

$$\frac{V_2}{j4} = \frac{15.80\angle93.23^\circ}{4\angle90^\circ} = 3.95\angle3.23^\circ \text{ A flowing away from node 2}$$

(c) The current in the  $5 \Omega$  resistance is given by:

$$I_5 = \frac{V_1 - V_2}{5} = \frac{17.12\angle-5.30^\circ - 15.80\angle93.23^\circ}{5}$$

$$\text{i.e., } I_5 = \frac{(17.05 - j1.58) - (-0.89 + j15.77)}{5}$$

$$= \frac{17.94 - j17.35}{5} = \frac{24.96\angle-44.04^\circ}{5}$$

$$= 4.99\angle-44.04^\circ \text{ A flowing from node 1 to node 2}$$

(d) The active power dissipated in the  $2.5 \Omega$  resistor is given by

$$P_{2.5} = (I_{2.5})^2(2.5) = \left(\frac{V_2 - 25\angle90^\circ}{2.5}\right)^2 (2.5)$$

$$= \frac{(0.89 + j15.77 - j25)^2}{2.5} = \frac{(9.273\angle-95.51^\circ)^2}{2.5}$$

$$= \frac{85.99\angle-191.02^\circ}{2.5} \text{ by de Moivre's theorem}$$

$$= 34.4\angle169^\circ \text{ W}$$