UNIT IV TRANSIENT ANALYSIS

TRANSIENT RESPONSE FOR DC CIRCUITS

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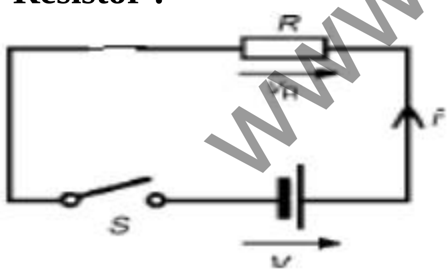
1. INTRODUCTION

For higher order differential equation, the number of arbitrary constants equals the order of the equation. If these unknowns are to be evaluated for particular solution, other conditions in network must be known. A set of simultaneous equations must be formed containing general solution and some other equations to match number of unknown with equations.

We assume that at reference time t=0, network condition is changed by switching action. Assume that switch operates in zero time. The network conditions at this instant are called initial conditions in network.

1. Resistor :

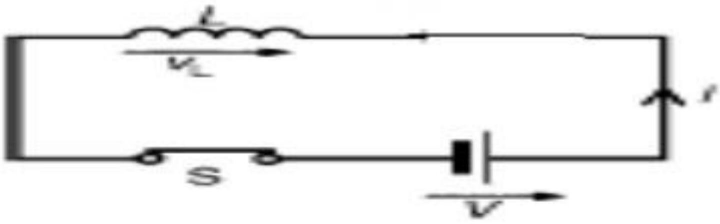


$$V = i \times R \longrightarrow (1)$$

Equ 1 is linear and also time dependent. This indicates that current through resistor changes if applied voltage changes instantaneously. Thus in resistor, change in current is instantaneous as there is no storage of energy in it.

2.Inductor:

$$V_L = L \bullet \frac{diL}{dt} \longrightarrow (2)$$



If dc current flows through inductor, dil/dt becomes zero as dc current is constant with respect to time. Hence voltage across inductor, VL becomes zero. Thus, as for as dc quantities are considered, in steady stake, inductor acts as short circuit.

We can express inductor current in terms of voltage developed across it as

$$iL = \frac{1}{L}\int V_L dt$$

In above eqn. The limits of integration is from $-\infty$ to $t(0^-)$.

Assuming that switching takes place at t=0, we can split limits into two intervals as

$$i_L = \frac{1}{L}\int_{-\infty}^{t} V_L dt$$

$$i_L = \frac{1}{L}\int_{-\infty}^{0-} V_L dt + \frac{1}{L}\int_{0^-}^{t} V_L dt$$

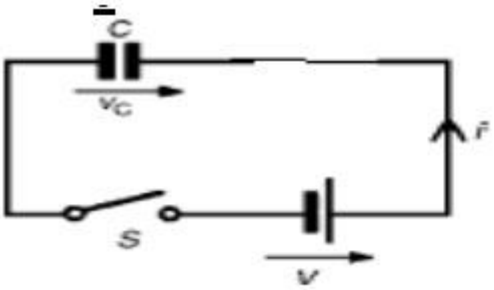
$$i_L = i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} V_L dt$$

at t = 0^t we can write $i_L(0^+) =$ $i_L(0^-) + \frac{1}{L}\int V_L dt$

$i_L(0^+) = i_L(0^-)$

I through inductor cannot change instantaneously.

3.capacitor



$$i_C = C\frac{dV_C}{dt}$$

If dc voltage is applied to capacitor, *dVC / dt* becomes zero as dc voltage is constant with respect to time.

Hence the current through capacitor iC becomes zero, Thus as far as dc quantities are considered capacitor acts as open circuit.

$$V_C = \frac{1}{C}\int i_C dt$$

$$V_C = \frac{1}{C}\int_{-\infty}^{t} i_C dt$$

Splitting limits of integration

$$V_C = \frac{1}{C}\int_{-\infty}^{0-} i_C dt + \frac{1}{C}\int_{0^-}^{t} i_C dt$$

At t(0^+), equation is given by

$$V_C(0^+) = V_C(0^-) + \frac{1}{C}\int_{0-}^{0+} i_C dt$$

$$V_C(0^+) = V_C(0^-)$$

Thus voltage across capacitor can not change instantaneously.

2. TRANSIENT RESPONSE OF RL CIRCUITS:

So far we have considered dc resistive network in which currents and voltages were independent of time. More specifically, Voltage (cause input) and current (effect output) responses displayed simultaneously except for a constant multiplicative factor (VR). Two basic passive elements namely, inductor and capacitor are introduced in the dc network. Automatically, the question will arise whether or not the methods developed in lesson-3 to lesson-8 for resistive circuit analysis are still valid. The voltage/current relationship for these two passive elements are defined by the derivative (voltage across the inductor

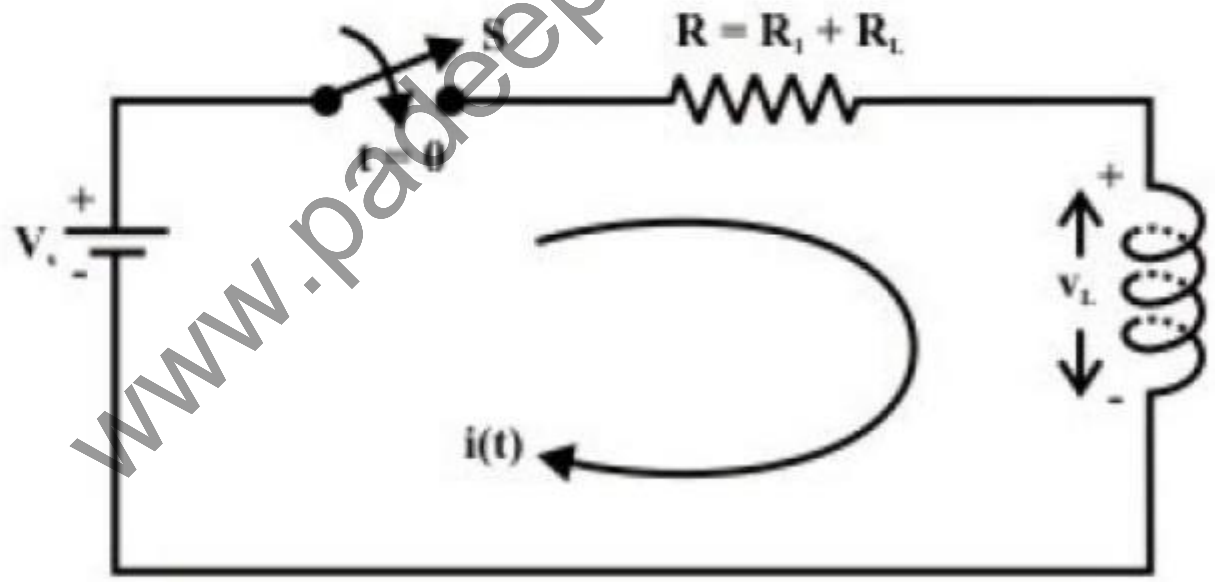
$$v_L(t) = L\frac{di_L(t)}{dt}$$

where =current flowing through the inductor ; current through the capacitor

$$i_C(t) = C\frac{dv_C(t)}{dt},$$

voltage across the capacitor) or in integral form as ()C

$$i_L(t) = \frac{1}{L}\int_0^t v_L(t)dt + i_L(0) \quad or \quad v_C(t) = \frac{1}{C}\int_0^t i(t)dt + v_C(0)$$



Our problem is to study the growth of current in the circuit through two stages, namely; (i) dc transient response (ii) steady state response of the system

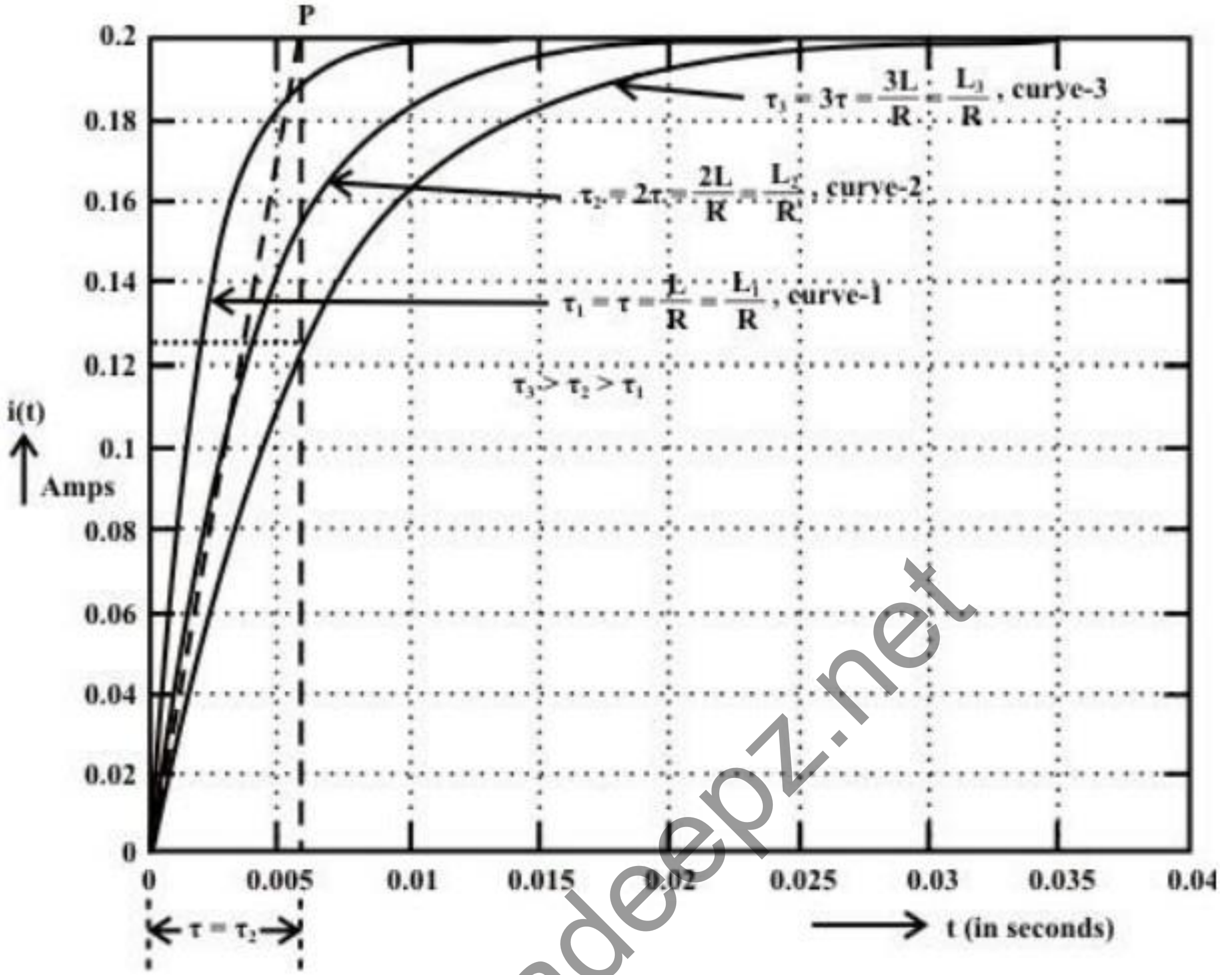
D.C Transients: The behavior of the current and the voltage in the circuit switch is closed until it reaches its final value is called dc transient response of the concerned circuit. The response of a circuit (containing resistances, inductances, capacitors and switches) due to sudden application of voltage or current is called transient response. The most common instance of a transient response in a circuit occurs when a switch is turned on or off –a rather common event in an electric circuit.

Growth or Rise of current in R-L circuit

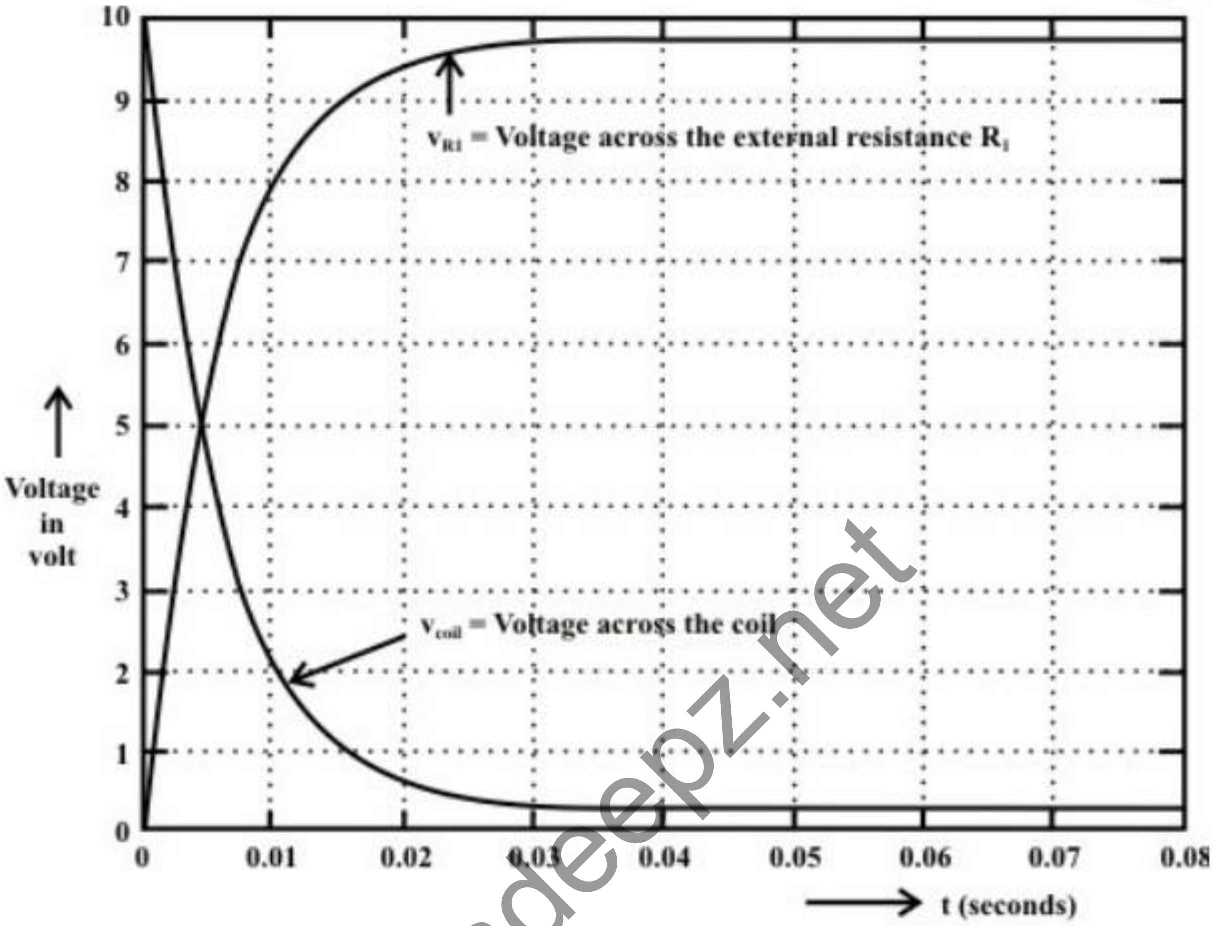
To find the current expression (response) for the circuit shown in fig. 10.6(a), we can write the KVL equation around the circuit

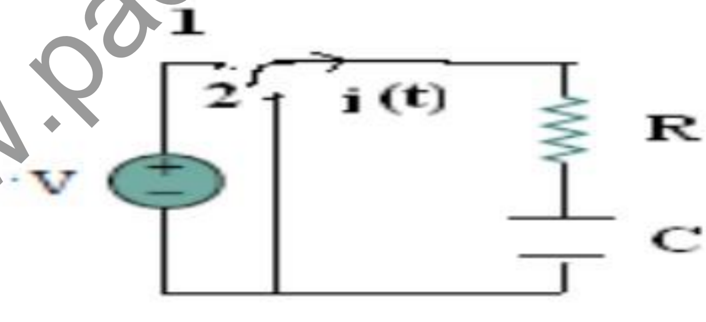
The table shows how the current i(t) builds up in a R-L circuit.

Actual time (t) in sec	Growth of current in inductor (Eq.10.15)
$t=0$	$i(0)=0$
$t=\tau\left(=\frac{L}{R}\right)$	$i(\tau)= 0.632 \times\frac{V_s}{R}$
$t=2\tau$	$i(2\tau)= 0.865 \times\frac{V_s}{R}$
$t=3\tau$	$i(3\tau)= 0.950 \times\frac{V_s}{R}$
$t=4\tau$	$i(4\tau)= 0.982 \times\frac{V_s}{R}$
$t=5\tau$	$i(5\tau)= 0.993 \times\frac{V_s}{R}$



P
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
i(t)
Amps
$\tau_3 = 3\tau = \frac{3L}{R} = \frac{L_3}{R}$, curve-3
$\tau_2 = 2\tau = \frac{2L}{R} = \frac{L_2}{R}$, curve-2
$\tau_1 = \tau = \frac{L}{R} = \frac{L_1}{R}$, curve-1
$\tau_3 > \tau_2 > \tau_1$
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
$\tau = \tau_2$
t (in seconds)





Consider network shown in fig. the switch k is moved from position 1 to 2 at reference time t = 0.

Now before switching take place, the capacitor C is fully charged to V volts and it discharges through resistance R. As time passes, charge and hence voltage across capacitor i.e. Vc decreases gradually and hence discharge current also decreases gradually from maximum to zero exponentially.

After switching has taken place, applying kirchoff's voltage law,

$$0 = V_R + V_C$$

Where VR is voltage across resistor and VC is voltage across capacitor.

$$\therefore V_C = -V_R = -i \cdot R$$

$$i = C\frac{dv_C}{dt}$$

$$\therefore V_C = -R \cdot C \cdot \frac{dv_C}{dt}$$

Above equation is linear, homogenous first order differential equation. Hence rearranging we have,

$$\frac{dt}{RC} = -\frac{dv_C}{V_C}$$

Integrating both sides of above equation we have

$$\frac{t}{RC} = -\ln V_C + k'$$

Now at t = 0, VC =V which is initial condition, substituting in equation we have,

$$\therefore 0 = -\ln V + k'$$

$$\therefore k' = \ln V$$

Substituting value of k' in general solution, we have

$$\frac{t}{RC} = -\ln V_C + \ln V$$

$$\therefore \frac{t}{RC} = \ln\frac{V}{V_C}$$

$$\therefore \frac{V}{V_C} = e^{\frac{t}{RC}}$$

$$\therefore V_C = V \cdot e^{-\frac{t}{RC}}$$

$$V = \frac{Q}{C}$$

Where Q is total charge on capacitor

Similarly at any instant, VC = q/c where q is instantaneous charge.

So we have, $\frac{q}{c} = \frac{Q}{C}e^{-\frac{t}{RC}}$

$$q = Q \cdot e^{-\frac{t}{RC}}$$

Thus charge behaves similarly to voltage across capacitor.
Now discharging current i is given by

$$i = \frac{V_C}{R}$$

but $V_R = V_C$ when there is no source in circuit.

$$\therefore i = \frac{V_C}{R}$$

$$\therefore i = \frac{V}{R} e^{-\frac{t}{RC}}$$

but VR = VC when there is no source in circuit.

The above expression is nothing but discharge current of capacitor. The variation of this current with respect to time is shown in fig.

This shows that the current is exponentially decaying. At point P on the graph. The current value is (0.368) times its maximum value. The characteristics of decay are determined by values R and C, which are 2 parameters of network.

For this network, after the instant t = 0, there is no driving voltage source in circuit, hence it is called undriven RC circuit.

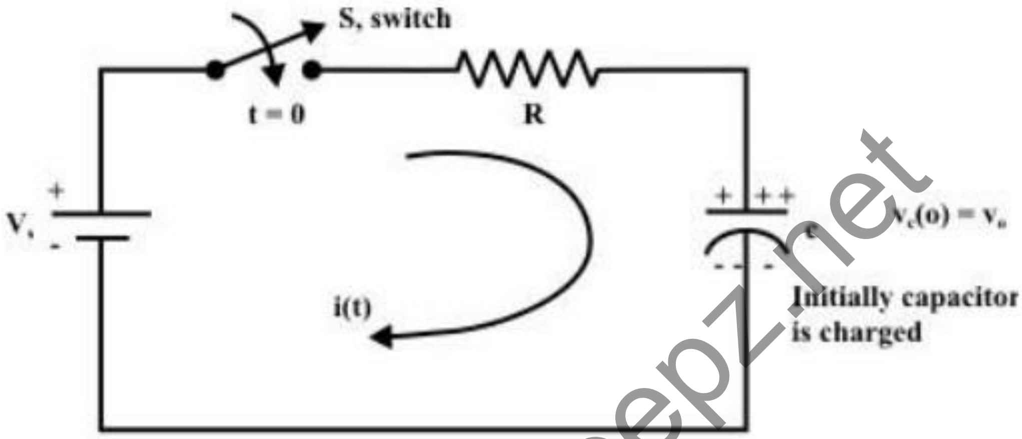
3. TRANSIENT RESPONSE OF RC CIRCUITS

Ideal and real capacitors: An ideal capacitor has an infinite dielectric resistance and plates (made of metals) that have zero resistance. However, an ideal capacitor does not exist as all dielectrics have some

leakage current and all capacitor plates have some resistance. A capacitor's of how much charge (current) it will allow to leak through the dielectric medium. Ideally, a charged

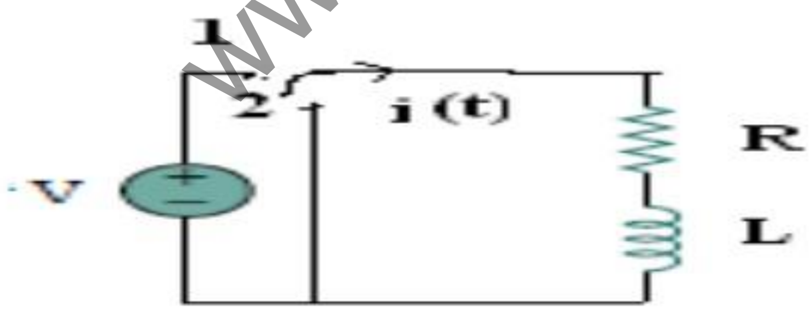
capacitor is not supposed to allow leaking any current through the dielectric medium and also assumed not to dissipate any power loss in capacitor plates resistance. Under this situation, the model as shown in fig. 10.16(a) represents the ideal capacitor. However, all real or practical capacitor leaks current to some extend due to leakage resistance of dielectric medium. This leakage resistance can be visualized as a resistance connected in parallel with the capacitor and power loss in capacitor plates can be realized with a resistance connected in series with capacitor. The model of a real capacitor is shown in fig.

Let us consider a simple series RC−circuit shown in fig. 10.17(a) is connected through a switch 'S' to a constant voltage source .



The switch 'S' is closed at time 't=0' It is assumed that the capacitor is initially charged with a voltage and the current flowing through the circuit at any instant of time '' after closing the switch is

<u>Current decay in source free series RL circuit:</u> -



At t = 0- , , switch k is kept at position 'a' for very long time. Thus, the network is in steady state. Initial current through inductor is given as,

$$i_L(0^-) = I_0 = \frac{V}{R} = i_L(0^+) \text{ ------------------------ } 1$$

Because current through inductor can not change instantaneously

Assume that at t = 0 switch k is moved to position 'b'.
Applying KVL,

$$L\frac{di}{dt} + iR = 0 \quad \text{-----------------------------} \; 2$$

$$\therefore L\frac{di}{dt} = -iR$$

Rearranging the terms in above equation by separating variables

$$\frac{di}{i} = -\frac{R}{L}dt$$

Integrating both sides with respect to corresponding variables

$$\therefore \ln(i) = -\frac{R}{L} \quad \text{---------------------------} \; 3$$

Where k’ is constant of integration.
To find- k’:
Form equation 1, at t=0, i=I0
Substituting the values in equation 3

$$\therefore \ln(I_0) = -\frac{R}{L}(0) + k'$$

$$k' = \ln(I_0) \quad \text{--------------------} \; 4$$

Substituting value of k’ from equation 4 in

$$\ln(i) = -\frac{R}{L}t + \ln(I_0)$$

$$\ln(i) - \ln(I_0) = -\frac{R}{L}t$$

$$\frac{i}{i_0} = e^{-\frac{R}{L}t}$$

$$\therefore i = I_0 \cdot e^{-\frac{R}{L}t} \quad \text{----------------------}5$$

fig. shows variation of current i with respect to time

From the graph, H is clear that current is exponentially decaying. At point P on graph. The current value is (0.363) times its maximum value. The characteristics of decay are determined by values R and L which are two parameters of network.
The voltage across inductor is given by

$$V_L = L\frac{di}{dt} = L\frac{d}{dt}\left[I_0 \cdot e^{-\frac{R}{L}t}\right] = L \cdot I_0\left(-\frac{R}{L}\right) \cdot e^{-\frac{R}{L}t}$$

$$\therefore V_L = -I_0 \cdot \mathrm{Re}^{-\frac{R}{L}t}$$

But $I_0 \cdot R = V$

$$\therefore V_L = -V \cdot e^{-\frac{R}{L}t} \quad \text{Volts}$$

4. TRANSIENT RESPONSE OF RLC CIRCUITS

In the preceding lesson, our discussion focused extensively on dc circuits having resistances with either inductor () or capacitor () (i.e., single storage element) but not both. Dynamic response of such first order system has been studied and discussed in detail. The presence of resistance, inductance, and capacitance in the dc circuit introduces at least a second order differential equation or by two simultaneous coupled linear first order differential equations. We shall see in next section that the complexity of analysis of second order circuits increases significantly when compared with that encountered with first order circuits. Initial conditions for the circuit variables and their derivatives play an important role and this is very crucial to analyze a second order dynamic system.

Response of a series R-L-C circuit

Consider a series RLcircuit as shown in fig.11.1, and it is excited with a dc voltage source C−−sV.
Applying around the closed path for ,

$$L\frac{di(t)}{dt}+Ri(t)+v_c(t)=V_s$$

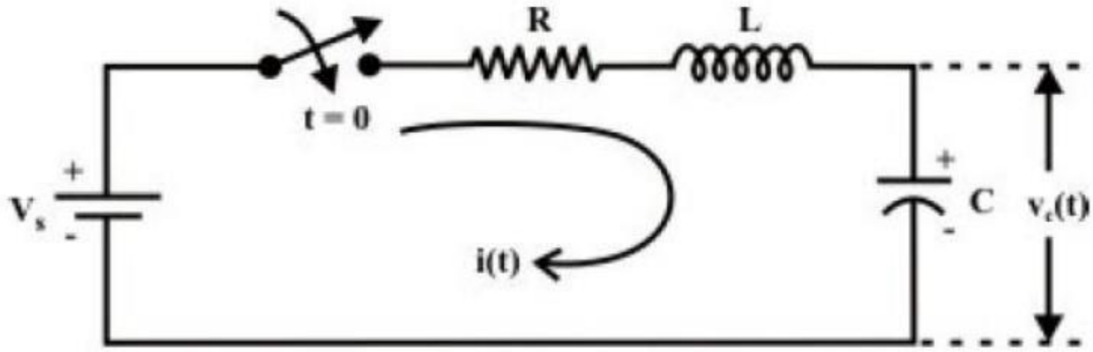


Fig. 11.1: A Simple R-L-C circuit excited with a dc voltage source

The current through the capacitor can be written as Substituting the current ‘’expression in eq.(11.1) and rearranging the terms,

$$i(t)=C\frac{dv_c(t)}{dt}$$

$$LC\frac{d^2v_c(t)}{dt^2}+RC\frac{dv_c(t)}{dt}+v_c(t)=V_s$$

The above equation is a 2nd-order linear differential equation and the parameters associated with the differential equation are constant with time. The complete solution of the above differential equation has two components; the transient response and the steady state response. Mathematically, one can write the complete solution as

$$v_c(t)=v_{cn}(t)+v_{cf}(t)=\left(A_1\,e^{\alpha_1 t}+A_2\,e^{\alpha_2 t}\right)+A$$

$$LC\frac{d^2v_c(t)}{dt^2}+RC\frac{dv_c(t)}{dt}+v_c(t)=0\Rightarrow\frac{d^2v_c(t)}{dt^2}+\frac{R}{L}\frac{dv_c(t)}{dt}+\frac{1}{LC}v_c(t)=$$

$$a\,\frac{d^2v_c(t)}{dt^2}+b\,\frac{dv_c(t)}{dt}+c\,v_c(t)=0\ \text{(where } a=1,\ b=\frac{R}{L}\ \text{and}\ c=\frac{1}{LC}\text{)}$$

Since the system is linear, the nature of steady state response is same as that of forcing function (input voltage) and it is given by a constant value. Now, the first part of the total response is completely dies out with time while and it is defined as a transient or natural response of the system. The natural or transient response (see Appendix in Lesson-10) of second order differential equation can be obtained from the homogeneous equation (i.e., from force free system) that is expressed by

$$\alpha^2 + \frac{R}{L}\alpha + \frac{1}{LC} = 0 \quad \Rightarrow a\,\alpha^2 + b\,\alpha + c = 0 \text{ (where } a = 1,\ b = \frac{R}{L} \text{ and } c = \frac{1}{LC}$$

and solving the roots of this equation (11.5) on that associated with transient part of the complete solution (eq.11.3) and they are given below.

$$\alpha_1 = \left(-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \right) = \left(-\frac{b}{2a} + \frac{1}{a}\sqrt{\left(\frac{b}{2}\right)^2 - ac} \right);$$

$$\alpha_2 = \left(-\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \right) = \left(-\frac{b}{2a} - \frac{1}{a}\sqrt{\left(\frac{b}{2}\right)^2 - ac} \right)$$

where, $b = \frac{R}{L}$ and $c = \frac{1}{LC}$.

The roots of the characteristic equation are classified in three groups depending upon the values of the parameters ,,RLand of the circuit

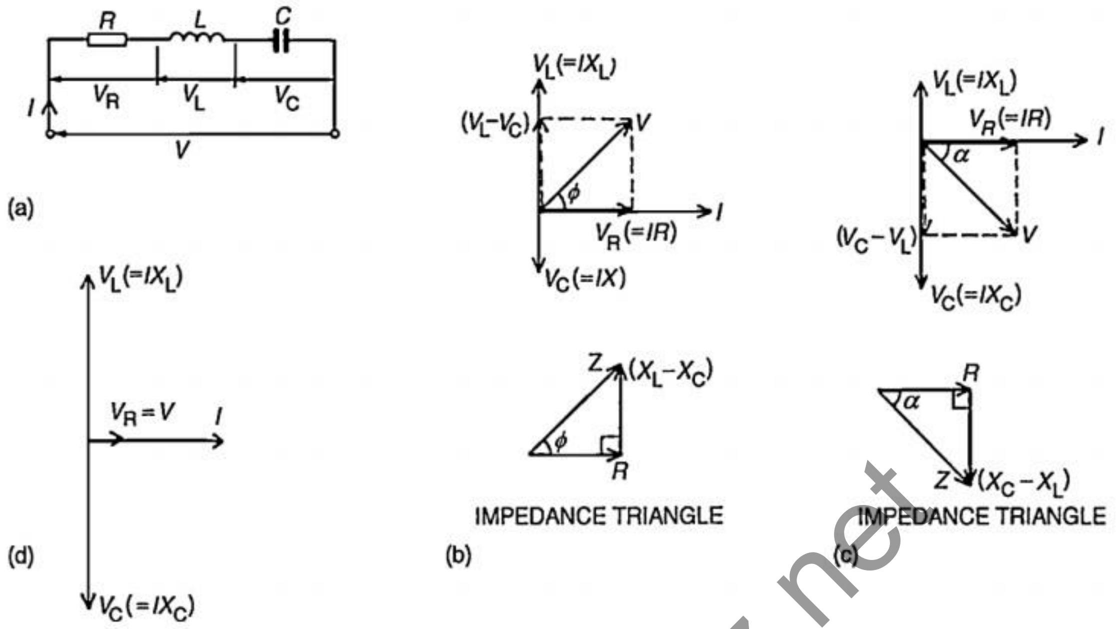
Case-A (overdamped response): That the roots are distinct with negative real parts. Under this situation, the natural or transient part of the complete solution is written as

$$v_{cn}(t) = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t}$$

and each term of the above expression decays exponentially and ultimately reduces to zero as and it is termed as overdamped response of input free system. A system that is overdamped responds slowly to any change in excitation. It may be noted that the exponential term t→∞11tAeαtakes longer time to decay its value to zero than the term21tAeα. One can introduce a factorξ that provides an information about the speed of system response and it is defined by damping ratio

$$(\xi) = \frac{Actual\ \ damping}{critical\ \ damping} = \frac{b}{2\sqrt{ac}} = \frac{R/L}{2/\sqrt{LC}} > 1$$

RLC Circuit:



Consider a circuit in which R, L, and C are connected in series with each other across ac supply as shown in fig.

The ac supply is given by, V = Vm sin wt

The circuit draws a current I. Due to that different voltage drops are,

1. Voltage drop across Resistance R is VR = IR

2. Voltage drop across Inductance L is VL = IXL

3. Voltage drop across Capacitance C is Vc = IXc The characteristics of three drops are,

(i) VR is in phase with current I

(ii) VL leads I by 900

(iii) Vc lags I by 900

According to krichoff's laws

<u>Steps to draw phasor diagram:</u>

1. Take current I as reference
2. VR is in phase with current I

3. VL leads current by 900
4. Vc lags current by 900

5. obtain resultant of VL and Vc. Both VL and Vc are in phase opposition (1800 out of phase)

6. Add that with VRby law of parallelogram to get supply voltage.

The phasor diagram depends on the condition of magnitude of VL and Vc which ultimately depends on values of XL and Xc.

Let us consider different cases:

Case(i): XL > Xc

When X L > Xc

Also VL > Vc (or) IXL > IXc

So, resultant of VL and Vc will directed towards VL i.e. leading current I. Hence I lags V i.e. current I will lags the resultant of VL and Vc i.e. (V L - Vc). The circuit is said to be inductive in nature.

From voltage triangle,

$$V = \sqrt{(V_R^2 + (V_L - Vc)^2)} = \sqrt{((IR)^2 + (IX_L - IXc)^2)}$$
$$V = I\sqrt{(R^2 + (X_L - Xc)^2)}$$
$$V = IZ$$
$$Z = \sqrt{(R^2 + (X_L - Xc)^2)}$$

If , V = Vm Sin wt ; i = Im Sin (wt - ɸ)

i.e I lags V by angle ɸ

Case(ii): XL < Xc

When XL < Xc

Also VL < Vc (or) IXL < IXc

Hence the resultant of VL and Vc will directed towards Vc i.e current is said to be capacitive in nature

Form voltage triangle

$$V = \sqrt{(V_R^2 + (V_c - V_L)^2)} = \sqrt{((IR)^2 + (IX_c - IX_L)^2)}$$
$$V = I\sqrt{(R^2 + (X_c - X_L)^2)}$$
$$V = IZ$$
$$Z = \sqrt{(R^2 + (Xc - X_L)^2)}$$

If , V = Vm Sin wt ; i = Im Sin (wt + ɸ)

i.e I lags V by angle ɸ

i.e I lags V by angle ɸ

Case(iii): XL = Xc

When XL = Xc

Also VL = Vc (or) IXL = IXc

So VL and Vc cancel each other and the resultant is zero. So V = VR in such a case, the circuit is purely resistive in nature.

Impedance:

In general for RLC series circuit impedance is given by, Z = R + j X
X = XL –Xc = Total reactance of the circuit
If XL > Xc ; X is positive & circuit is Inductive
If XL < Xc ; X is negative & circuit is Capacitive
If XL = Xc ; X =0 & circuit is purely Resistive
Tan ϕL - =Xc[(X)⁄R]
Cos ϕ = [R⁄Z]
Z = 2 √+(X L(R-Xc) 2)

Impedance triangle:

In both cases R = Z Cos ϕ

X = Z Sin ϕ

Power and power triangle:

The average power consumed by circuit is,

Pavg = (Average power consumed by R) + (Average power consumed by L) + (Average power consumed by C)

Pavg = Power taken by R = I2R = I(IR) = VI

V = V Cos ϕ P = VI Cos ϕ

Thus, for any condition, XL > Xc or XL < Xc General power can be expressed as

P = Voltage x Component in phase with voltage

Power triangle:

S = Apparent power = I2Z = VI

P = Real or True power = VI Cos ϕ = Active po Q = Reactive power = VI Sin ϕ

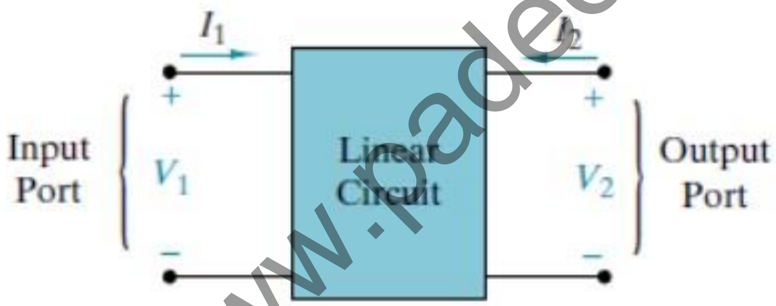
5. CHARACTERIZATION OF TWO PORT NETWORKS IN TERMS OF Z,Y AND H PARAMETERS.

The purpose of this appendix is to study methods of characterizing and analyzing two-port networks. A port is a terminal pair where energy can be supplied or extracted. A two-port network is a four-terminal circuit in which the terminals are paired to form an input port and an output port. Figure W2–1 shows the customary way of defining the port voltages and currents. Note that the reference marks for the port variables comply with the passive sign convention.

The linear circuit connecting the two ports is

Assumed to be in the zero state and to be free of any independent sources. In other words, there is no initial energy stored in the circuit and the box in Figure W2–1 contains only resistors, capacitors, inductors, mutual inductance, and dependent sources. A four-terminal network qualifies as a two-port if the net current entering each terminal pair is zero. This means that the current exiting the lower port terminals in Figure W2–1 must be equal to the currents entering the upper terminals.

One way to meet this condition is to always connect external sources and loads between the input terminal pair or between the output terminal pair. The first task is to identify circuit parameters that characterize a two-port. In the two port approach the only available variables are the port voltages V1 and V2, and the port currents I1 and I2. A set of two-port parameters is defined by expressing two of these four-port variables in terms of the other two variables. In this appendix we study the four ways in Table W2–1.



TWO-PORT PARAMETERS

NAME	EXPRESS	IN TERMS OF	DEFINING EQUATIONS
Impedance	V_1, V_2	I_1, I_2	$V_1 = z_{11}I_1 + z_{12}I_2$ and $V_2 = z_{21}I_1 + z_{22}I_2$
Admittance	I_1, I_2	V_1, V_2	$I_1 = y_{11}V_1 + y_{12}V_2$ and $I_2 = y_{21}V_1 + y_{22}V_2$
Hybrid	V_1, I_2	I_1, V_2	$V_1 = h_{11}I_1 + h_{12}V_2$ and $I_2 = h_{21}I_1 + h_{22}V_2$
Transmission	V_1, I_1	$V_2, -I_2$	$V_1 = A\,V_2 - B\,I_2$ and $I_1 = C\,V_2 - D\,I_2$

Note that each set of parameters is defined by two equations, one for each of the two dependent port variables. Each equation involves a sum of two terms, one for each of the two independent port variables. Each term involves a proportionality because the two-port is a linear circuit and superposition applies. The names given the parameters indicate their dimensions (impedance and admittance), a mixture of dimensions (hybrid), or their original application (transmission lines). With double-subscripted parameters, the first subscript indicates the port at which the dependent variable appears and the second subscript the port at which the independent variable appears.

Regardless of their dimensions, all two-port parameters are network functions. In general, the parameters are functions of the complex frequency variable and s-domain circuit analysis applies. For sinusoidalsteady-state problems, we replace s by j and use phasor circuit analysis. For purely resistive circuits, the two-port parameters are real constants and we use resistive circuit analysis. Before turning to specific parameters, it is important to specify the objectives of two-port network analysis. Briefly, these objectives are:

1. Determine two-port parameters of a given circuit.

Use two-port parameters to find port variable responses for specified input sources and output loads.

In principle, the port variable responses can be found by applying node or mesh analysis to the internal circuitry connecting the input and output ports. So why adopt the two-port point of view? Why not use straightforward circuit analysis?

There are several reasons. First, two-port parameters can be determined experimentally without resorting to circuit analysis. Second, there are applications in power systems and microwave circuits in which input and output ports are the only places that signals can be measured or observed. Finally, once two-port parameters of a circuit are known, it is relatively simple to find port variable responses for different input sources and/or different output loads.

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IMPEDANCE PARAMETERS

The impedance parameters are obtained by expressing the port voltages V1 and V2 in terms of the port currents I1 and I2.

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2 \qquad \text{(W2–1)}$$

The network functions z11, z12, z21, and z22 are called the impedance parameters or simply the z-parameters. The matrix form of these equations are

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \text{(W2–2)}$$

where the matrix [z] is called the impedance matrix of a two-port network. To measure or compute the impedance parameters, we apply excitation at one port and leave the other port open-circuited. When we drive port 1 with port 2 open (I2), the expressions in Eq. (W2–1) reduce to one term each, and yield the definitions of z11 and z21.

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \text{input impedance with the output port open} \qquad \text{(W2–3a)}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \text{forward transfer impedance with the output port open}$$

Conversely, when we drive port 2 with port 1 open (I1=0), the expressions in Eq. (W2–1) reduce to one term each that define z12and z22 as

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \text{reverse transfer impedance with the input port open} \qquad \text{(W2–4a)}$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \text{output impedance with the input port open} \qquad \text{(W2–4b)}$$

All of these parameters are impedances with dimensions of ohms. A two-port is said to be reciprocal when the open-circuit voltage measured at one port due to a current excitation at the other port is unchanged when the measurement and excitation ports are interchanged. A two-port that fails this test is said to be nonreciprocal. Circuits containing resistors, capacitors, and inductors (including mutual inductance) are always reciprocal. Adding dependent sources to the mix usually makes the two-port nonreciprocal. If a two-port is reciprocal, then z12 z21. To prove this we apply an excitation I1 Ix at the input port and observe that Eq. (W2–1) gives the open circuit (I2) voltage at the output port as V2OC z21Ix. Reversing the excitation and observation ports, we find that an excitation I2 Ix produces an open-circuit (I1) voltage at the input port of V 1OC z12Ix. Reciprocity requires that V1OC
V2OC, which can only happen if z12 z21.

ADMITTANCE PARAMETERS

The admittance parameters are obtained by expressing the port currents I1 and I2 in terms of the port voltages V1 and V2. The resulting two-port i–v relationships are

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2 \qquad \text{(W2–5)}$$

The network functions y11, y12, y21, and y22 are called the admittance parameters or simply the y-parameters. In matrix form these equations are

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [\mathrm{y}] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \text{(W2–6)}$$

where the matrix [y] is called the admittance matrix of a two-port network. To measure or compute the admittance parameters, we apply excitation at one port and short circuit the other port. When we drive at port 1 with port 2 shorted (V2= 0), the expressions in Eq. (W2–5) reduce to one term each that define y11 and y21 as

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \text{input admittance with the output port shorted} \qquad \text{(W2–7a)}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \text{forward transfer admittance with the output port shorted}$$

Conversely, when we drive at port 2 with port 1 shorted (V1= 0), the expressions in Eq. (W2– 5) reduce to one term each that define y22 and y12 as

$$\left.\frac{I_1}{V_2}\right|_{V_1=0} = \text{reverse transfer admittance with the input port shorted}$$

$$\text{(W2–8a)}$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \text{output admittance with the input port shorted} \qquad \text{(W2–8b)}$$

All of these network functions are admittances with dimensions of Siemens. If a two-port is reciprocal, then y12 y21. This can be proved using the same process applied to the z-parameters.

The admittance parameters express port currents in terms of port voltages, whereas the impedance parameters express the port voltages in terms of the port currents. In effect these parameters are inverses. To see this mathematically, we multiply Eq. (W2–2) by [z] 1, the inverse of the impedance matrix.

$$[\mathbf{z}]^{-1}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [\mathbf{z}]^{-1}[\mathbf{z}]\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

In other words,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [\mathbf{z}]^{-1}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

HYBRID PARAMETERS

The hybrid parameters are defined in terms of a mixture of port variables. Specifically, these parameters express V1 and I2 in terms of I1 and V2. The resulting two-port i–v relationships are

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \qquad \text{(W2–9)}$$

Where h11, h12, h21, and h22 are called the hybrid parameters or simply the h-parameters. In matrix form these equations are

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [\mathbf{h}]\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \qquad \text{(W2–10)}$$

Where the matrix [h] is called the h-matrix of a two-port network. The h-parameters can be measured or calculated as follows. When we drive at port 1 with port 2 shorted (V2= 0), the expressions in Eq. (W2–9) reduce to one term each, and yield the definitions of h11 and h21.

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \text{input impedance with the output port shorted} \qquad \text{(W2–11a)}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \text{forward current transfer function with the output port shorted} \qquad \text{(W2–11b)}$$

When we drive at port 2 with port 1 open (I1=0), the expressions in Eq. (W2–9) reduce to one term each, and yield the definitions of h12 and h22.

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \text{reverse voltage transfer function with the input port open} \qquad \text{(W2–12a)}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \text{output admittance with the input port open} \qquad \text{(W2–12b)}$$

These network functions have a mixture of dimensions: h11 is impedance in ohms, h22 is admittance in Siemens, and h21 and h12 are dimensionless transfer functions. If a two-port is reciprocal, then h12h21. This can be proved by the same method applied to the z-parameters.

AC through pure resistance:

Consider a simple circuit consisting of a pure resistance 'R' ohms across voltage

V = VmSinwt

According to ohms law,

i = V/R = (VmSinwt)/R

i = (Vm/R) Sin(wt)

This is equation giving instantaneous value of current

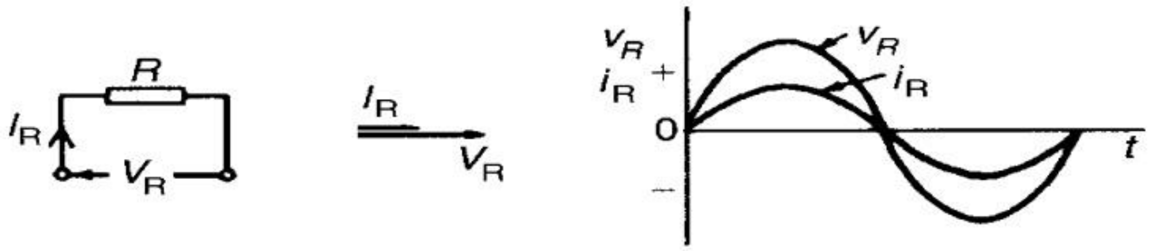
i = ImSin(wt+ϕ)

Im = Vm/R and ϕ= 0

It is in phase with the voltage applied. There is no phase different between two.

"In purely resistive circuit, the current and the voltage applied are in phase with each other **"**

Ac through purely resistive circuit:



Circuit diagram   Phasor diagram   Current and voltage waveforms

Power:

The instantaneous power in a.c circuit can be obtained by taking product of the instantaneous value of current and voltage.

P= VxI
= Vm Sin(wt) x Im Sin(wt)
= VmIm Sin^2wt
=(VmIm⁄2)x(1-coswt)

P= (VmIm⁄2) – (VmIm⁄2)coswt

Instantaneous power consists of two components: 1- Constant power component (VmIm⁄2)

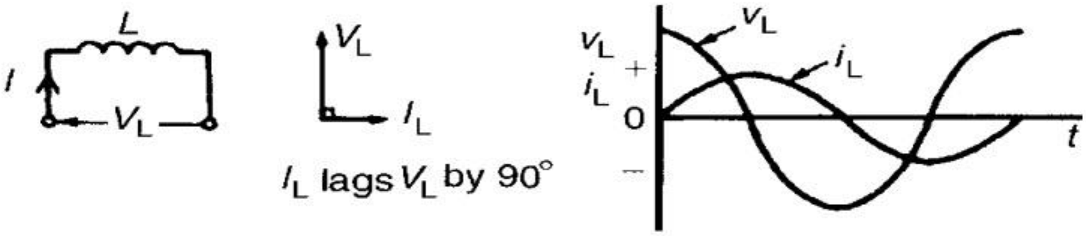
2- Fluctuating component [(VmIm⁄2)coswt]having frequency, double the frequency of applied voltage.

The average value of fluctuating cosine component of double frequency is zero, overone complete cycle. So, average power consumption over one cycle is equal to constant power component i.e. VmIm⁄2.

Pavg = VmIm⁄2 = (Vm⁄√2) x (Im⁄√2)
Pavg = Vrms x Irms watts

Pavg = VxI watts =I^2R watt

AC through pure inductance:



Circuit diagram

Phasor diagram

Current and voltage waveforms

Consider a simple circuit consisting of a pure, inductance of L henries connected across a voltage given by the equation.

$$V = Vm \ Sinwt$$

Pure inductance has zero ohmic resistance its internal resistance is zero. The coil has pure inductance of h henries (H).

When alternating current 'i' flows through inductance 'L'. It sets up an alternating magnetic field around the inductance. This changing flux links the coil and due to self inductance emf gets induced in the coil. This emf opposes the applied voltage.

The self induced emf in the coil is given by Self induced emf e= -L di/dt

At all instants, applied voltage V is equal and opposite to self induced emf e

$$V = -e = -(-L \ di/dt)$$
$$V = L \ di/dt$$
$$Vm \ Sinwt = L \ di/dt$$
$$di = (Vm/L) \ Sinwt \ dt$$
$$i = \int di = \int (Vm/L) \ Sinwt \ dt$$
$$=(Vm/L)[-coswt/w]$$

$$i = -(Vm/wL) \ Sin((\pi/2)-wt) \qquad \Rightarrow coswt = Sin(wt-\pi/2)$$

$i= -(Vm/wL)\ Sin\ (wt-\pi/2)$ ⇨ $Sin\ ((\pi/2)-wt) =-Sin(wt-\pi/2)$

$$\mathbf{i = Im\ Sin\ (wt- \pi/2)}$$

Where, $Im = Vm/wL = Vm/X_L$

$$\mathbf{X_L = wL = 2\pi fL\ \Omega}$$

The above equation clearly shows that the current is purely sinusoidal and having angle of - $\pi/2$ radians i.e. 90^o. This means current lags voltage applied by 90^o

Concepts of Induction Reactance:

$Im = Vm/X_L$ Where, $X_L = wL = 2\pi fL$ $\mathbf{\Omega}$

X_L = Induction Reactance

Inductive reactance is defined as the opposition offered by the inductance of circuit to the flow of an alternating sinusoidal current.

Note:

If frequency is zero, which is so for dc voltage, the inductive reactance is zero. Therefore it is said that inductance offers zero reactance for dc or steady current.

Power:

$P = VxI$

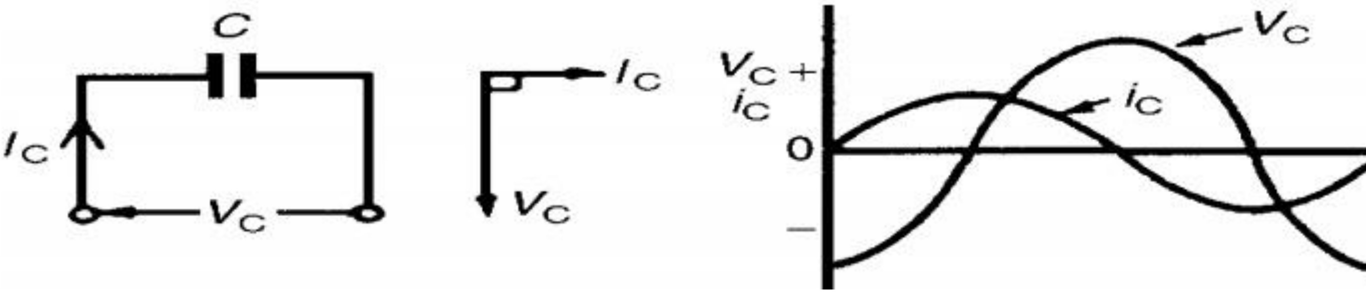
$= Vm\ Sinwt\ x\ Im\ sin(wt- \pi/2)$

$= -VmImSin(wt)\ Cos(wt)$ $[\because\ sin(wt- \pi/2)=-Cos(wt)]$

$P = (-VmIm/2)\ x\ Sin(2wt)$ $[\because 2sinwt\ Coswt = Sin2wt]$

The average value of Sine curve over a complete cycle is always zero. $Pav = \int_0^{2\pi} Sin\ (2wt)\ d(wt)$

AC through pure capacitance:



Consider a simple circuit consisting of pure capacitor of c farads, connected across a voltage given by equation,

$$V = Vm \text{ Sin}wt$$

The current I charge the capacitor C. The instantaneous charge 'q' on the plates of capacitor is given by

$$q= CV$$
$$q = CV_m \sin wt$$

Current i = rate of flow of charge 'q' i = dq/dt = d(CVmSinwt)/dt

$$i = CVm\ d(\text{Sin}wt)/dt$$

$$i = Vm/(1/wc) \text{ Sin}(wt+\pi/2) \quad i = Im \text{ Sin}(wt+\pi/2)$$

Where, Im =Vm/Xc

Xc = 1/wc = 1/(2πfc) Ω

The above equation clearly shows that current is purely sinusoidal and having phase angle of π/2 radians $+90^0$

This means current leads voltage applied by 90^0. The positive sign indicates leading nature of the current.

Concepts of reactive capacitance:

Im=Vm/Xc And Xc=1/wC = 1/(2πfc) Ω

Xc = Capacitive reactance

Capacitive reactance is defined as the opposition offered by the capacitance of the circuit to flow of an alternating sinusoidal current.

Power:

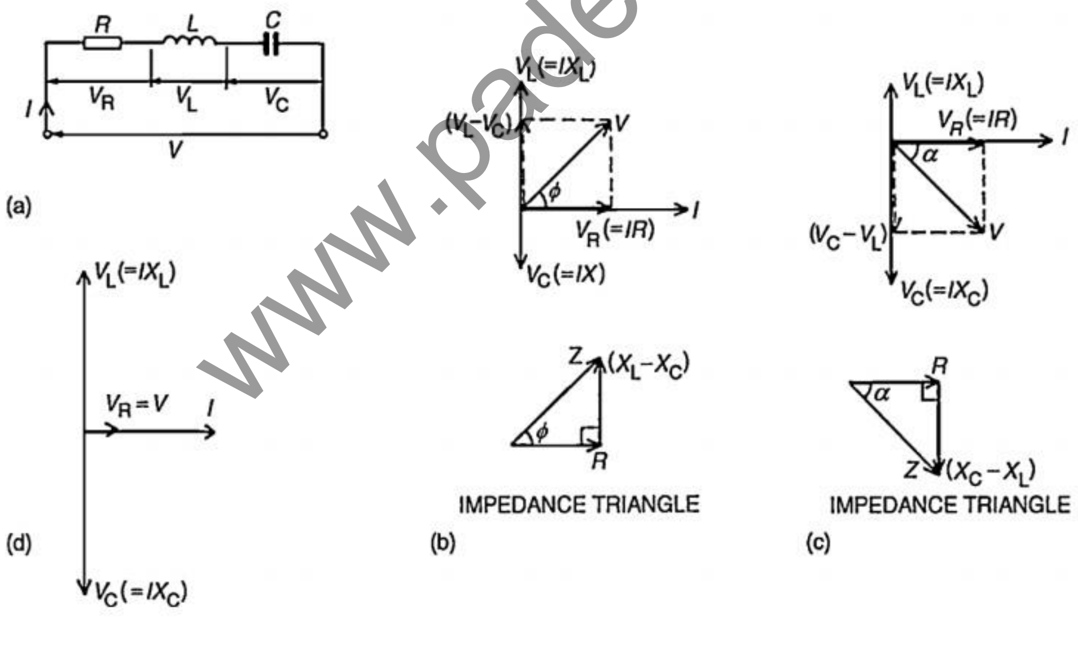
The expression for instantaneous power can be obtained by taking the product of instantaneous voltage and current

$$P = V \times i = V_m \sin(wt) \times I_m \sin(wt + \pi/2)$$
$$= V_m I_m \sin(wt) \cos(wt)$$
$$P = (V_m I_m/2) \sin(2wt)$$
$$P_{avg} = P_{av} = \int_0^{2\pi} (V_m I_m/2) \sin(2wt)\, d(wt) = 0$$

Drawing of the phasor diagram for a series RLC circuit energized by a sinusoidal voltage showing the relative position of current, component voltage and applied voltage for the following case

a) **When $X_L > X_c$**
b) **When $X_L < X_c$**
c) **When $X_L = X_c$.**

RLC Circuit:



Consider a circuit in which R, L, and C are connected in series with each other across ac supply as shown in fig.

The ac supply is given by,

$$V = V_m \sin wt$$

The circuit draws a current I. Due to that different voltage drops are,

1) Voltage drop across Resistance R is $V_R = IR$
2) Voltage drop across Inductance L is $V_L = IX_L$
3) Voltage drop across Capacitance C is $Vc = IXc$ The characteristics of three drops are,

1. V_R is in phase with current I
2. V_L leads I by 90^0
3. Vc lags I by 90^0

According to krichoff's laws

Steps to draw phasor diagram:

1. Take current I as reference
2. V_R is in phase with current I
3. V_L leads current by 90^0
4. Vc lags current by 90^0
5. obtain resultant of V_L and Vc. Both V_L and Vc are in phase opposition (180^0 out of phase)
6. Add that with VRby law of parallelogram to getsupply voltage.

The phasor diagram depends on the condition of magnitude of V_L and Vc which ultimately depends on values of X_L and Xc.

Let us consider different cases:

Case(i): $X_L > Xc$

When $X_L > Xc$

Also $V_L > Vc$ (or) $IX_L > IXc$

So, resultant of V_L and Vc will directed towards V_L i.e. leading current I. Hence I lags V i.e. current I will lags the resultant of V_L and Vc i.e. (V_L - Vc). The circuit is said to be inductive in nature.

From voltage triangle,

$V = \sqrt{(V_R^2 + (V_L - Vc)^2)} = \sqrt{((IR)^2 + (IX_L - IXc)^2)}$

$V = I\sqrt{(R^2 + (X_L - Xc)^2)}$

$V = IZ$

$Z = \sqrt{(R^2 + (X_L - Xc)^2)}$

If , $V = Vm \sin wt$; $i = Im \sin (wt - \phi)$

i.e I lags V by angle ϕ

Case(ii): $X_L < Xc$

When $X_L < Xc$

Also $V_L < Vc$ (or) $IX_L < IXc$

Hence the resultant of V_L and Vc will directed towards Vc i.e current is said to be capacitive in nature Form voltage triangle

$$V = \sqrt{(V_R^2 + (V_c - V_L)^2)} = \sqrt{((IR)^2 + (IX_c - IX_L)^2)}$$
$$V = I\sqrt{(R^2 + (X_c - X_L)^2)}$$
$$V = IZ$$
$$Z = \sqrt{(R^2 + (Xc - X_L)^2)}$$

If , V = Vm Sin wt ; i = Im Sin (wt + ф)

i.e I lags V by angle ф

i.e I lags V by angle ф

Case(iii): X_L = Xc

When X_L = Xc

Also V_L = Vc (or) IX_L = IXc

So V_L and Vc cancel each other and the resultant is zero. So V = V_R in such a case, the circuit is purely resistive in nature.

Impedance:

In general for RLC series circuit impedance is given by,

Z = R + j X

X = X_L – Xc = Total reactance of the circuit

If XL > Xc ; X is positive & circuit is Inductive

If XL < Xc ; X is negative & circuit is Capacitive

If XL = Xc ; X =0 & circuit is purely Resistive

Tan ф = [(X_L - Xc)/R]

Cos ф = [R/Z]

Z = $\sqrt{(R^2 + (X_L - Xc)^2)}$

Impedance triangle:

In both cases R = Z Cos ф

X = Z Sin ф

Power and power triangle:

The average power consumed by circuit is,

Pavg = (Average power consumed by R) + (Average power consumed by L) + (Average power consumed by C)

Pavg = Power taken by R = I^2R = I(IR) = VI

$V = V \cos \phi$

$P = VI \cos \phi$

Thus, for any condition, $X_L > Xc$ or $X_L < Xc$ General power can be expressed as

P = Voltage x Component in phase with voltage

Power triangle:

S = Apparent power = $I^2Z = VI$

P = Real or True power = $VI \cos \phi$ = Active power

Q = Reactive power = $VI \sin \phi$

1. An alternating current of frequency 60Hz has a maximum value of 12A

1. **Write down value of current for instantaneous values**
2. **Find the value of current after 1⁄360 second**
3. **Time taken to reach 9.6A for the first time.**

In the above cases assume that time is reckoned as zero when current wave is passing through zero and increase in positive direction.

<u>**Solution**</u>:

Given:

F = 60Hz

Im = 12A

$W = 2\pi f = 2\pi \times 60 = 377$ rad⁄sec

(i). Equation of instantaneous value is $i = Im \sin wt$

$i = 12 \sin 377t$

(ii). $t = 1/360$sec

$i = 12 \sin (377/360) = 12 \sin 1.0472 = 10.3924$ A

i = 10.3924 A

(iii). $i = 9.6$ A

$9.6 = 12 \sin 377t$ $\sin 377t = 0.8$ $377t = 0.9272$

2. A 50 Hz, t = 2.459 x 10^{-3}sec alternating voltage of 150V (rms) is applied independently

a. **Resistance of 10Ω**
b. **Inductance of 0.2H**

c. Capacitance of 50uF

Find the expression for the instantaneous current in each case. Draw the phasor diagram in each case.

<u>Solution</u>:

Given ,

F = 50Hz

V = 150 V

$$Vm = \sqrt{2}\ Vrms = \sqrt{2}\ x\ 150 = 212.13\ V$$

Case (i):

R = 10 Ω

Im = Vm⁄R = 212.13⁄10 = 21.213

For pure resistive current circuit phase different Φ

Φ = 0

i = Im Sin wt = Im Sin 2πft

Im = Vm⁄R = 212.13⁄10 = 21.213

ϕ = 0

For pure resistive current circuit phase different ϕ

i = Im Sin wt = Im Sin 2πft

i = 21.213 Sin (100 πt) A

Phasor diagram: Case (ii):

L = 0.2H

X_L = wL =2πfL X_L = 2π x 50 x0.2

X_L = 62.83 Ω

Im = Vm⁄X_L = 212.13⁄62.83 = 3.37A

Φ A = -90^0 = π⁄2 rad

In pure Inductive circuit, I lags V by 90 degree i= Im Sin (wt -) A

i = 3.37 Sin Φ (wt - Φ) A

i = 3.37 Sin (100 πt - π⁄2) A

Phasor diagram

Case(iii):

C= 50uf

Xc = 1⁄wC = 1⁄2πfC

$X_c = 1/(2\pi \times 50 \times 50 \times 10^{-6}) = 63.66\Omega$

Im = Vm⁄Xc = 212.13⁄63.66 = 3.33 A

In pure capacitive circuit, current leads voltage by 900

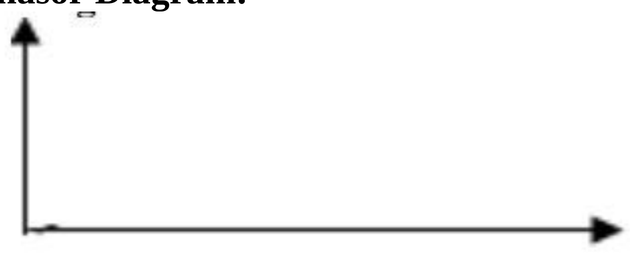
= 90o = π⁄2 rad

i = Im Sin (wt +ϕ) A

i = 3.33 Sin (wt +ϕ) A

i = 3.33 Sin (100 πt + π⁄2) A

Phasor Diagram:



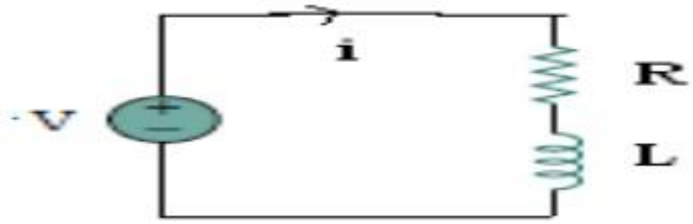
3. An alternating current i= 414 Sin (2π x 50 x t) A is passed through a series circuit of a resistance of 100 ohm and an inductance of 0.31831 H. find the expression for the instantaneous values of voltage across,

a. The resistance ,

b. Inductance

c. Capacitance

Solution:



Given

$i = 414 \sin (2\pi \times 50\, t)$ A

$R = 100\ \Omega$

$L = 0.31831$ H

$X_L = 2\pi \times 50 \times 0.31831 = 100\ \Omega$

(i) **<u>Voltage across Resistance:</u>**

$V_R = iR = 1.414 \sin (2\pi \times 50\, t) \times 100$

$V_R = 141.4 \sin (2\pi \times 50\, t)$ V

R.m.s value of $V_R = 141.4/\sqrt{2} = 100$ V

$\Phi = 0°$

$V_R = 100\angle 0° = 100 + j0$ V

(ii) <u>Voltage across Inductance:</u>

$V_L = i\, X_L = 1.414 \sin (2\pi \times 50\, t + 90^0) \times 100$

$V_L = 141.4 \sin (2\pi \times 50\, t + 90^0)$ V

$$\text{R.m.s value of } V_L = 141.4/\sqrt{2} = 100 \text{ V}, \ \Phi = 90°$$

$$V_L = 100\angle 90° = 0 - j100 \text{ V}$$

$$V = V_R + V_L = 100 + j\,0 + 0 + 100j$$

$$V = 100 + j100 = 141.42\angle 45^0 \text{ V}$$

$$Vm = \sqrt{2} \text{ x } 141.42 = 200 \text{ V}$$

$$V = 200 \text{ Sin } (2\pi \text{ x } 50\,t + 45^0) \text{ V}$$

4. The wave form of the voltage and current of a circuit are given by
e= 120 Sin (314 t)
i= 10 Sin (314 t + π/6)

Calculate the value of resistance, capacitance which is connected in series to form the circuit. Also, Draw wave forms for current, voltage and phasor diagram. Calculate power consumed by the circuit.

<u>Solution:</u>

Given:

V = 120 Sin (314 t) ; Vm = 120 V; $2\pi f = 314$; f = 50 Hz
i = 10 Sin (314 + $\pi/6$) ; Im = 10 A ; $\Phi = 30°$

$V = Vm/\sqrt{2} = 120/\sqrt{2} = 84.85$ V
$I = Im/\sqrt{2} = 10/\sqrt{2} = 7.07$ A
$|Z| = V/I = 84.45/7.07 = 12\ \Omega$
$Z = 12\angle -30° \rightarrow$ As current leads by $30°$
Ckt is RC series Circuit is capacitive nature.
Φ = -Ve
$Z = 12\angle -30° = 10.393 - j6\Omega$ use P $\rightarrow$ R
Z = R – j Xc
R = 10.393 ; Xc = 6 Ω
Xc = $1/2\pi fC$
6 = $1/(2\pi$ x50 x C)
C = 530.45 uF
P = VI Cos Φ = 84.85 x 7.07 xCos 30^0
P = 519.52

5. A resistance of 120 Ω and a capacitive reactance of 250Ω are connected in series across a AC voltage source. If a current 0.9 A is flowing in the circuit find out,
(i). Power factor
(ii). Supply voltage
(iii). Voltage across resistance and capacitance
(iv). Active power and reactive power
Solution:
Given :
R = 120Ω

$Xc = 250\,\Omega$

$I = 0.9$ A.

$Z = R - j\,Xc = 120 - 250j = 277.308 - 64.358$

Power factor:

$Cos\Phi = Cos(-64.358) = 0.4327 Leading$

Supply Voltag:

$V = IZ = 0.9 \times 277.308 \angle -64.358°$

$V = 249.5772 \angle -64.358°$ V

V_R and Vc:

$V_R = IR\ 0.9 \times 120 = 108$ V

$Vc = I\,Xc = 0.9 \times 250 = 225$ V

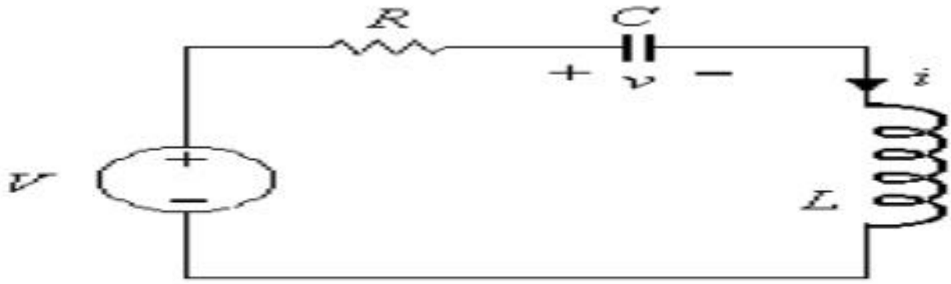
Active power and Reactive power:

$P = VI\,Cos\ \Phi = 249.5772 \times 0.9 \times 0.4327 = 97.1928$ W

$Q = VI\,Sin\ \Phi = 247.5772 \times 0.9 \times Sin(-64.358^0) = -202.498$ VAR

- Ve Sign $\rightarrow$ Leading nature

6. A series circuit consisting of 25 Ω resistor, 64mH inductor and 80uF capacitor to a 110V, 50Hz, Single phase supply as shown in fig. Calculate the current, Voltage across individual element and overall p.f of the circuit. Draw a neat phasor diagram showing



Solution:

$R = 25\,\Omega$

$L = 64mH$

$C = 80uF$

$X_L = 2\pi fL = 2\pi \times 50 \times 64 \times 10^{-3} = 20.10\ \Omega$

$Xc = 1/2\pi fC = 1/(2\pi \times 50 \times 80 \times 10^{-6}) = 39.78\,\Omega$

$Z = R + j\,X_L - j\,Xc$

$= 25 + j\,20.10 - j\,39.78$

$Z = (25 - j\,19.68)\ \Omega$

$I = V/Z = {110\angle 0^\circ}/{25 - j19.68} = 110\angle 0^\circ/(31.81\angle -38.20^\circ)$

$Z = (25 - j\,19.68)$

$I = V/Z = 3.4580\ A$

$V_R = IR\,(3.4580\angle 38.20^\circ)(25) = 86.45\angle 38.20^\circ\ V$

$V_L = I\,j\,X_L = (3.4580\angle 38.20^\circ)(j\,20.10)$

$= (3.4580\angle 38.20^\circ)(20.10\angle 90^\circ$

$V_L = 69.50\ \angle 128.12^\circ\ V$

$Vc = I(-j\,Xc) = (3.4580\angle 38.20^\circ)(-\,j\,39.78)$

$= (3.4580\angle 38.20^\circ)(38.78\angle -90^\circ)$

$= 134.10\ \angle -15.9^\circ V$

$V = 110\angle 0^\circ$ Volts

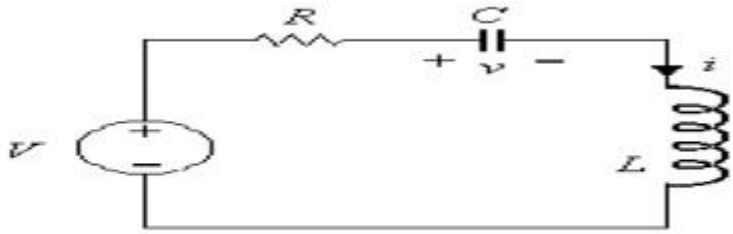
$Cos\Phi = Cos38.20^\circ = 0.7858$ Leading

7. A series circuit having pure resistance of 40 , pure inductance of 50.07mH and a capacitance is connected across a 400V, 50Hz Ac supply. This R, L, C combination draws a current of 10A. Calculate

1. **Power factor of circuit**
2. **Capacitor value**

Solution:

Solution:

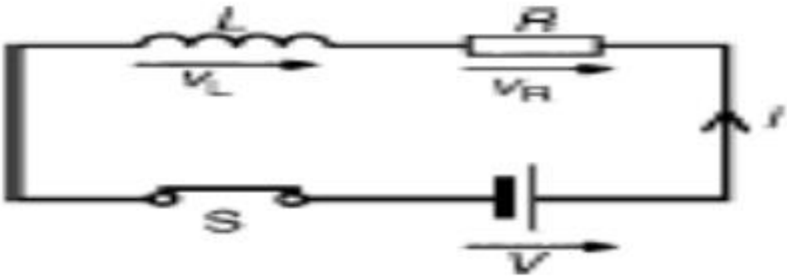


Given:
$R = 40\,\Omega$
$L = 50.07\text{mH}$
$C = ?.$
$V = 400\text{ V}$
$f = 50\text{ Hz}$
$I = 10\text{ A}$
$XL = X_L = 2\pi fL = 2\pi \times 50 \times 50.07 \times 10^{-3} = 15.73\,\Omega$
$Xc = 1/2\pi fC$
$Z = \sqrt{R + j(XL - Xc)}$
$|Z| = \sqrt{(R^2 + (XL - Xc)^2)}$
$|Z| = |V|/|I| = 400/10 = 40\,\Omega$
$40 = \sqrt{(40^2 + (15.73 - Xc)^2)}$
$40^2 = 40^2 + (15.73 - Xc)^2$
$Xc = 15.73\ \Omega$
$1/2\pi fC = 15.73 \quad \Rightarrow \quad C = 1/(2\pi f \times 15.73)$
$C = 2.023 \times 10^{-4}\text{ F}$
$Z = 40 + j(15.73 - 15.73) = 40 + j0\ \Omega$
$Z = 40\,\angle 0°\Omega$
Power factor $\text{Cos}\Phi = \text{Cos}\,0° = 1$.

1) What is time constant? Explain time constant in case of series RL circuit.
Or
2) **A series RL circuit with initial current I_0 in the inductor is connected to a dc voltage V at t = 0. Derive the expression for instantaneous current through the Inductor for t>0.**
Or
3) Explain in brief about the step response of series RL circuits.

The response or the output of the series RL and RC circuits driven dc excitations is called step response of the network.

Consider that a dc voltage is applied to any general network through a switch k as shown in fig.



Initially switch k is kept open for very long time. So no voltage is applied to the network. Thus the voltage at input-terminals of network is zero. So we can write voltage across terminals A and B V

(l) is zero. When the switch k is closed at t=0, the dc voltage v gets applied to the network. The voltage across terminals A and B suddenly or instantaneously rises to voltage V. the variation of voltage across terminals +1 and B against time t as shown in fig (b).

In fig (b) it is observed that at t=0, there is a step of V volts. Such signal or function is called step function. We can define step function as

$$V(t) = V; t \geq 0$$
$$= 0; t < 0$$

When the magnitude of the voltage applied is 1 volt then the function is called unit step function.

When the circuits are driven by driving sources, then such circuits are called driven circuits. When the circuits are without such driving sources, then such circuits are called undriven circuits or
source free circuits.

Step response of Driver series RL circuit:-

Consider a series RL circuit.
At $t=0^-$, switch k is about to close but not fully closed. As voltage is not applied to the circuit, current in the circuit will be zero.

$$i_L(0^-) = 0$$

In this current through inductor can not change instantaneously.

$$i_L(0^+) = 0$$

Let initial current through inductor can be represented as I_0. in above case I_0 is zero. Assume that switch k is closed at t =0.

Apply KVL:

$$V = IR + L\frac{di}{dt}$$

$$\frac{V}{R} = i + \frac{L}{R}\frac{di}{dt}$$

$$\frac{V}{R} - i = \frac{L}{R}\frac{di}{dt}$$

$$\frac{R}{L}dt = \frac{di}{\frac{V}{R} - i}$$

Integrating both sides,

$$\int\frac{R}{L}dt = \int\frac{di}{\frac{V}{R} - i}$$

$$\rightarrow \frac{R}{R}t = -\ln\left[\frac{V}{R} - i\right] + k'$$

k'= integration constant.

To find k':

At t = 0, i = I_0 = 0

$$\frac{R}{L}(0) = -\ln\left[\frac{V}{R} - 0\right] + k'$$

$$\ln\left[\frac{V}{R}\right] = k'$$

sub values of k' in eqn 2 we get

$$\frac{Rt}{L} = -\ln\left[\frac{V}{R} - i\right] + \ln\left[\frac{V}{R}\right]$$

$$\frac{R}{L}t = \ln\left[\frac{\frac{V}{R}}{\frac{V}{R} - i}\right]$$

Take anti log

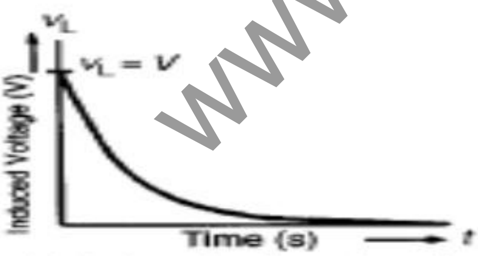
$$e^{\frac{R}{L}t} = \frac{\frac{V}{R}}{\frac{V}{R} - i}$$

$$\frac{V}{R} - i = \frac{V}{R}e^{\frac{-R}{L}t}$$

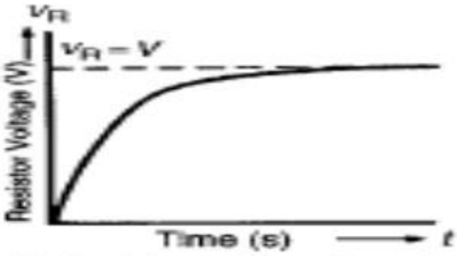
$$i = \frac{V}{R}\left[1 - e^{\frac{-R}{L}t}\right]$$

Term V/R = steady state current

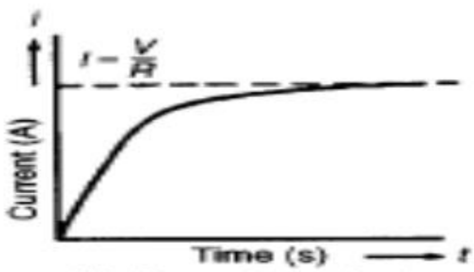
$\frac{-V}{R}e^{-\frac{R}{L}t} =$ Transient part of solution of current.



(a) Induced voltage transient



(b) Resistor voltage transient



(c) Current transient

From above fig (a) shows variation of current I with respect to time (t) i.e. current increases exponentially with respect to time. The rising current produces rising flux, which induces emf in coil. According to Lens's law, the self induced emf opposes the flow of current. Because of this induced emf and its opposition, the current in the coil don't reach its max value.

The point p shown on graph indicates that current in circuit rises to 0.632 time's maximum value of current in steady state.

"the time required for the current to rise to the 0.632 of its final value is known as time constant of given RL circuit. The time constant is denoted by z". Thus for series RL circuit, time constant is

$$\tau = \frac{R}{L}\sec$$

The initial rate of rise of current is large up to first time constant. At later stage, the rate of rise of current reduces.

Theoretically I reach maximum value after infinite time.
Voltage across inductor L is given by

$$V_L = L\frac{di}{dt}$$

$$V_L = L\frac{d}{dt}\left[\frac{V}{R}\left(1-e^{\frac{-R}{L}t}\right)\right]$$

$$V_L = L\left\{\frac{d}{dt}\left(\frac{V}{R}\right)-\frac{d}{dt}\left(\frac{V}{R}\bullet e^{-\frac{R}{L}t}\right)\right.$$

$$V_L = L\left\{0-\left(\frac{V}{R}\right)\left(-\frac{R}{L}\right)\left[e^{-\frac{R}{L}t}\right]\right\}$$

$$V_L = Ve^{-\frac{R}{L}t}\,volts$$