Steady State error of a System

Dr. Bishakh Bhattacharya

Professor, Department of Mechanical Engineering

IIT Kanpur

This Lecture Contains

- Steady state error of a system
- Position, Velocity and Acceleration Constants
- Optimal Steady State Error

Steady state error - Introduction

- **Steady state error refers to the long-term behavior of a dynamic system.**
- **The Type of a system is significant to predict the nature of this error.**
- **A system having no pole at the origin is referred as Type-0 system.**
- **Thus, Type-1, refers to one pole at the origin and so on.**
- **It will be shown in this lecture that, it is the type of a system which can directly determine whether a particular command will be followed by a system or not.**
- **We will consider three common commands: namely, step, ramp and parabolic ramp and find out the steady state response/error of a system to follow these commands.**
- **A closed loop control system shows remarkable performance in reducing the steady state error of a system.**

Steady state error of a system

- **Error in a system: E(s) = U(s) /(1+G(s))**

$$\lim_{t\to\infty} e(t) = e_{ss} = \lim_{s\to 0} \frac{sU(s)}{1+G(s)}$$

- **For a step input**

$$e_{ss} = \frac{A}{1+G(0)}$$

- **Plant Transfer function G(s) is defined as**

$$G(s) = \frac{K\prod_{i=1}^{M}(s+z_i)}{s^k \prod_{j=1}^{N}(s+p_j)}$$

Error Constants

- **Position error constant** $$K_p = \lim_{s \to 0} G(s)$$
- **Steady state error of a step input of magnitude A is** $e_{ss} = A/(1+K_p)$
- **Steady state error will be zero for system with type greater than or equal to 1**

- **For ramp input** $$e_{ss} = \lim_{s \to 0} \frac{A}{sG(s)}$$

- **Define velocity constant as** $$K_v = \lim_{s \to 0} sG(s)$$

- **Hence steady state error is** A/K_v

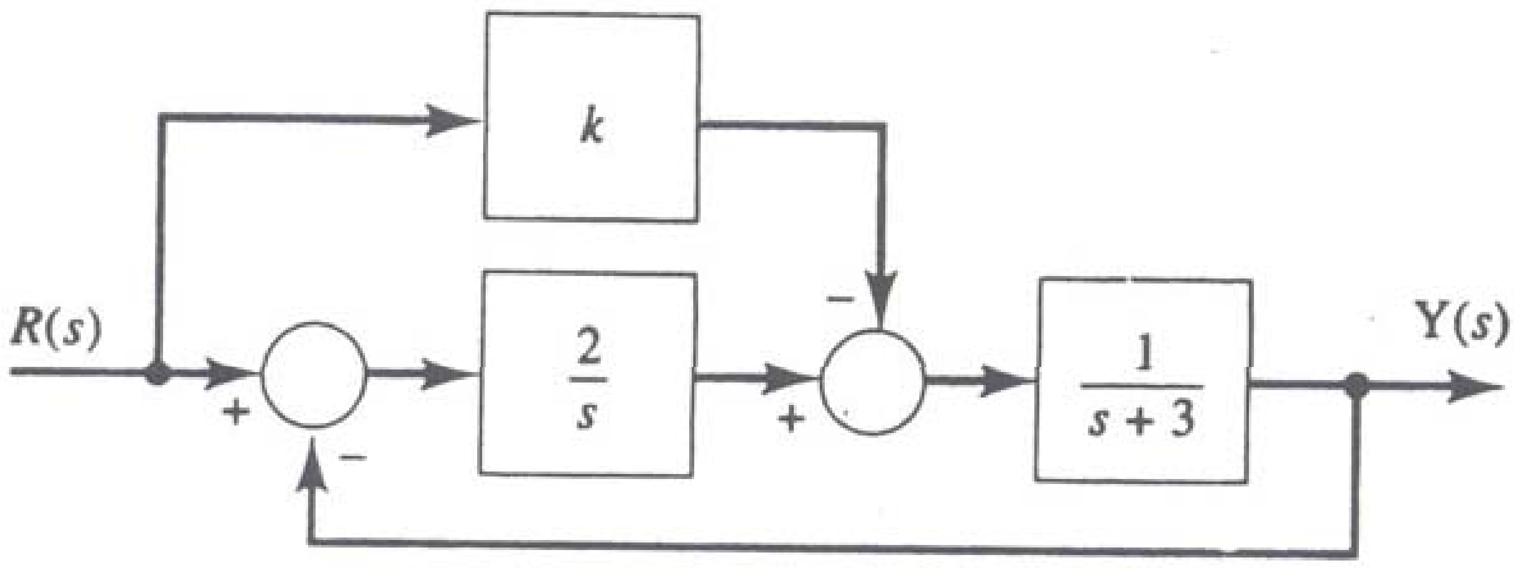
- **Error will be zero for k greater than or equal to 2**

Summary of Steady State Errors

Type	Step (A/s)	Ramp (A/s^2)	Parabolic Ramp (A/s^3)
0	$E_{ss} = A/(1+K_p)$	Inf	Inf
1	0	A/K_v	Inf
2	0	0	A/K_a

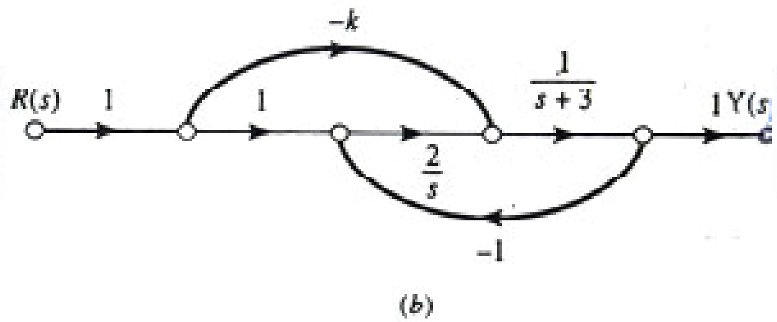
Design of Optimal systems

You can chose optimal gain such that the error constant could be minimized. Consider the following system:



(a)

Optimized Gain



$$T(s) = \frac{2 - ks}{s^2 + 3s + 2}$$

This is a Type zero system. Hence, consider step input.

$$E(s) = \frac{A}{s}(1 - T_e(s)) = A\left(\frac{k+2}{s+1} - \frac{k+1}{s+2}\right)$$

$$J(k) = \int_0^\infty e^2(t)dt = \frac{A^2}{12}[k^2 + 6k + 11]$$

Note that the Performance index is a quadratic function of gain k, which can be minimized to obtain k.

dJ/dk=0 --- k = -3

Special References for this lecture

- ***Feedback Control of Dynamic Systems:*** **Frankline, Powell and Emami-Naeini, Pearson Publisher**
- ***Control Systems Engineering:*** **Norman S Nise, John Wiley & Sons**
- ***Systems Dynamics and Response*: S. Graham Kelly, Thomson Publisher**