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Derivation Empirical Formula of Inductance Gradient of Rails for Circular Bore Geometry Using Regression Analysis Technique

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Abstract

The inductance gradient of the rails is important parameter in rail gun system as it directly decides the force acting on the armature. For the past few decades numeric methods and analytical methods are developed and used to calculate inductance gradient of the rails. Problems which are simple in nature are solved by using analytical methods and numerical methods are required codes and program and it is being considered as a time consuming process. Hence easy technique needs to calculate inductance gradient of rail. Since the inductance gradient value is purely depends on rail dimensions and its properties now researcher focused to obtain a simple formula which can be used to compute the L' value with respect to rail dimensions. In this paper an effort is made to obtain empirical formula that can be used to compute the L' value using curve fitting software called Oakdale date fit engineering. This software uses regression analysis to obtain the empirical formula for given set of data. Circular bore geometry is considered as a case study in this paper.

1. Introduction

The rail gun is an electromagnetic device that translates electrical energy which is supplied by energy storage devices into mechanical energy for accelerating the projectile which is either solid or plasma to hypervelocity. In order to accelerate the projectile between the conducting rails, it uses the magnetic field which is generated by the current that flows through two rails. The rail gun consists of permanently fixed two parallel conductors called the rails that are linked by a movable non ferromagnetic conductor called a projectile as shown Fig.1.

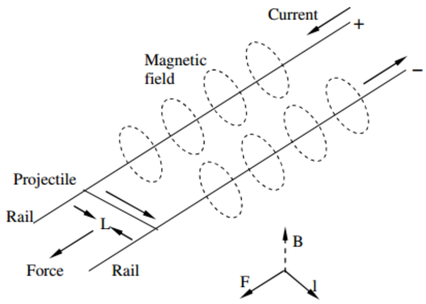

Fig. 1 A simple graphic representation of rail gun [9]

The force acting on the armature depends on inductance gradient of the rails. Inductance gradient of the rails plays an important role in the rail gun design as it determines the efficiency of the rail gun. Efficiency of the rail gun is defined as the ratio of projectile energy to input energy. To improve the efficiency of the rail gun, various geometry and dimensions of rails and armature are used. The Acceleration force and L' could be investigated instead of efficiency these are related by the following equation(1)

$$F = \frac{1}{2} L' I^2 \tag{1}$$

Where
L' is the inductance gradient of the rails in µH/m
I is the current through armature in Amps
F is the force acting on the armature in Newton

The calculation of inductance gradient is being considered as difficult as the current distribution is not uniform generally known as skin effect. Numerical method and analytical methods are used to calculate L' of the rails. For the past few decades various numerical methods and analytical methods are developed to calculate inductance gradient of the rails[2-6]. Since these two methods are being considered as time consuming methods now a day's researchers focusing to get simple empirical formula which can be used to calculate L' of the rails as its depends on rail dimension. In this paper, an attempt is made to extract an empirical formula that can be used to compute L' of the circular bore rails using regression analysis technique. Regression analysis is a statistical process for estimating the relationships among variables. It includes many techniques for modeling and analyzing several variables, when the focus is on the relationship between a dependent variable and one or more independent variables. The regression analysis technique uses the least square method to obtain the best fit of equations. The least square method assumes that the best fit curve for the given data has the minimal sum of the deviations squared error from the given set of data.[8]

Let us assume the given data points are $(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots (x_n, y_n)$

Where

x_i is the independent variable

y_i is the dependent variable

$f(x_i)$ is the fitting curve has the deviation error d_i from each data point and given as

$d_i = y_i - f(x_i)$

As per the least square method the best fitting curve has the property that

$$\Pi = d_1^2 + d_2^2 + d_3^2 + d_4^2 + \ldots\ldots d_n^2 = \sum_{i=1}^{n} d_i^2 = \sum_{i}^{n} \left[y_i - f(x_i) \right]^2 = \min imum \tag{2}$$

2. Empirical Equation Of Inductance Gradient Of Rail For Circular Geometry

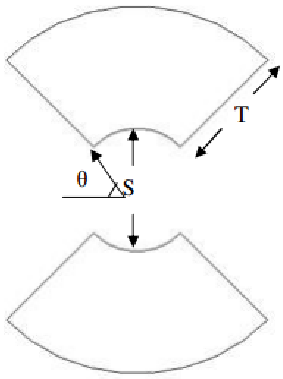

Fig.2 2-D view of circular bore geometry of rail gun

Fig.2 shows, the 2-D view of circular bore geometry of rail gun. The inductance gradient of the rail depends on rail dimensions of circular bore geometry such as rail separation (S), rail thickness (T), and opening angle of the rails (θ).An empirical formula which can be used to calculate L' of the rail has been obtained already using IEM methods for rectangular bore geometry. Obtaining the empirical formula to compute the inductance gradient value of the rails for rectangular geometry is easy as the three variables such as S, W, and H which can be converted into two variables such as S/H and W/H because the three variables are given in same unit i.e meter. But to get the empirical formula to compute L' of the rail for circular geometry is difficult as it has three different variables. The rail dimensions for circular bore is rail thickness (T), rail separation(S) and opening angle of the rails (θ). The T and S are measured in meter and θ is measured in degree, hence it is not able to convert the three variables into two variable problems. Initially all the three variables are varied and inductance gradient values are calculated using Finite element software named ANSOFT. These values are entered curve fitting software and simulated. But the best equation to compute L' value is not able to obtain. So it is decided to keep any one parameter of rail dimension of rail as a constant value. In this wok the opening angle of the rail is kept constant and rail thickness and rail separation are varied to calculate the L' values using ANSOFT field simulator[7].

2.1 L' formula for the opening angle 40 degree of the rails

Different ratios of rail thickness to rail separation (T/S) and rail separation are taken for calculating the L' values. The rail separation is varied from 3cm to 10cm in steps of 1cm. The ratio of (T/S) is varied from 0.1 to 1. Opening angle of the rail (θ) kept at 40 degree. For the opening angle 40 degree, inductance gradient values are calculated for varies values of T/S and S the values are tabulated in Table 1

Table 1 Inductance gradient value for various rail dimensions

		S							
S.No	T/S	10cm	9cm	8cm	7cm	6cm	5cm	4cm	3cm
1	1	0.3167	0.3205	0.3235	0.3271	0.3302	0.334	0.338	0.3442
2	0.9	0.3271	0.3304	0.3332	0.3368	0.34	0.3437	0.3481	0.3539
3	0.8	0.3384	0.3416	0.3444	0.348	0.351	0.3546	0.3591	0.3675
4	0.7	0.3516	0.3544	0.3574	0.3606	0.3638	0.3673	0.3718	0.378
5	0.6	0.3662	0.3693	0.3719	0.3759	0.3782	0.3818	0.3865	0.3932
6	0.5	0.3837	0.3861	0.389	0.3921	0.3953	0.399	0.404	0.4413
7	0.4	0.4044	0.4074	0.4099	0.4128	0.4162	0.4201	0.4253	0.4332
8	0.3	0.4309	0.4335	0.4361	0.4388	0.4425	0.4466	0.4522	0.4608
9	0.2	0.4648	0.467	0.4697	0.4728	0.4768	0.4814	0.4888	0.4976
10	0.1	0.5119	0.5147	0.517	0.5214	0.5251	0.5299	0.5353	0.5433

In Table 1the lower limit and upper limit of each constant value are given. In order to get accurate value of inductance gradient of the rails the constant values are chosen between lower limit and upper limit. The selected suitable constant values are given in the same table.

$$L' = a + b\ln\left(\frac{T}{S}\right) + C\left(\frac{1}{S}\right) + D\left(\ln\left(\frac{T}{S}\right)\right)^2 + e\left(\frac{1}{S^2}\right) + f\left(\frac{\ln\left(\frac{T}{S}\right)}{S}\right) +$$

$$g\left(\ln\left(\frac{T}{S}\right)\right)^3 + e\left(\frac{1}{S^3}\right) + h\left(\frac{\ln\left(\frac{T}{S}\right)}{S^3}\right) + i\left(\frac{(\ln\left(\frac{T}{S}\right))^2}{S}\right) \quad \mu\text{H/m} \qquad (3)$$

Where a , b , c , d ,e , f , g , h, i , j are constant. The constant values are given in Table 2

Table 2 Different values of constant

Variable	Lower Limit	Upper Limit	Best value of constants
a	0.25192	0.31046	0.28119
b	-0.10724	-6.9528653x10^{-02}	-0.088383
c	7.83636x10^{-02}	1.0017821	0.540073
d	-8.36485x10^{-03}	1.789995x10^{-02}	4.767554678x10^{-03}
e	-4.25042	0.3040510	-1.9731854
f	-0.181521	8.35541x10^{-02}	-4.898351x10^{-02}
g	-6.6924096x10^{-02}	6.432444055x10^{-02}	2.8816016x10^{-03}
h	-0.731095	6.2928088	2.780857
i	-0.286791	0.26171959	-0.012536
j	-4.70827x10^{-02}	7.1172846x10^{-02}	-1.99827292x10^{-02}

In Table 2 the lower limit and upper limit of each constant is given. In order to get accurate value the constant are value chosen between lower limit and upper limit and the best value is given in same table. The L' values obtained using empirical formula, for a different rail dimensions, are compared with FEM simulation and given in Table 3. From the table, it is observed that the L' values obtained using regression analysis equation shows a good agreement with value obtained using FEM method.

Table 3 Comparison of inductance gradient values with simulation values

S.No	Rail dimensions	L' from Ansoft field simulator (µH/m)	L' obtained from Regression analysis Empirical formula (µH/m)
1	T/S =1 , (θ) = 40 ° S = 9 cm	0.3205	0.3206
2	T/S =0.1 , (θ) = 40° S = 3cm	0.54423	0.54405

So for no one has given the L' equation for circular bore geometry, so the inductance gradient value obtained using regression analysis technique is compared only with the values obtained from the simulation. In order to analyze more detail about the equation, the graphs are plotted for various values of T/S and rail separation as shown in Figure 3 and Figure 4

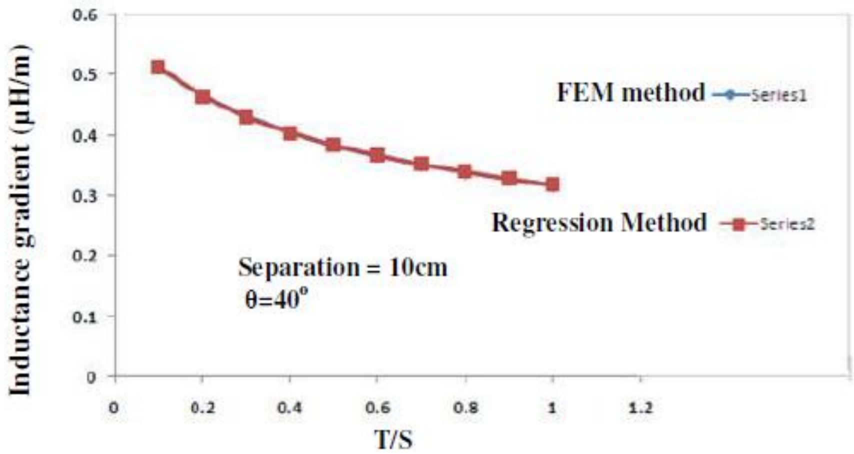

Figure 3 Inductance gradient values for various ratio of T/S for rail separation

S = 10cm

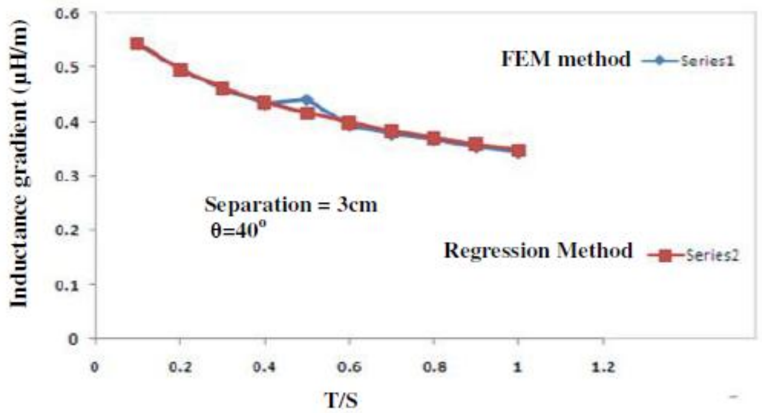

Figure 4 Inductance gradient values for various ratio of T/S for rail separation

S = 3cm

From the figure it is observed that the value obtained from the simulation and regression analysis equation shows good agreement.

2.2 .L' formula for the opening angle of the rails 10 degree

Different ratios of rail width to rail separation (T/S) , rail separation, are taken for calculating the L' value. The rail separation is varied from 3cm to 10cm in steps of 1cm. The ratio of (T/S) is varied from 0.1 to 1. For the opening angle 10 degree inductance gradient value is calculated varies value of T/S and S the values are tabulated given Table 4.

Table 4 inductance gradient value for various rail dimensions

S.No	T/S	S							
		10cm	9cm	8cm	7cm	6cm	5cm	4cm	3cm
1	1	0.1236	0.1249	0.1264	0.1277	0.1291	0.1314	0.1349	0.1399
2	0.9	0.1292	0.1304	0.1318	0.1329	0.1347	0.1371	0.1407	0.1459
3	0.8	0.1354	0.1367	0.1379	0.1396	0.1413	0.1437	0.1473	0.153
4	0.7	0.1428	0.1441	0.1455	0.147	0.1489	0.1516	0.1553	0.1613
5	0.6	0.1516	0.1528	0.1542	0.1559	0.1578	0.1609	0.1649	0.1714
6	0.5	0.1624	0.1636	0.1648	0.1671	0.1692	0.1725	0.1769	0.184
7	0.4	0.1758	0.1775	0.1793	0.1811	0.1838	0.1871	0.1919	0.1999
8	0.3	0.1943	0.1961	0.1979	0.2002	0.2029	0.2067	0.2122	0.2212
9	0.2	0.2203	0.2219	0.224	0.2266	0.2298	0.2341	0.2406	0.2507
10	0.1	0.2594	0.2615	0.264	0.267	0.2707	0.2754	0.2818	0.2916

The values obtained from the simulation are entered into curve fitting software and simulated. It gives 254 possible equations for the given data. Out of these 254 equations best one equation is chosen. The best equation to find the inductance gradient value of rail for the given data is given as

$$L' = a + b\ln\left(\frac{T}{S}\right) + C\left(\frac{1}{S}\right) + D\left(\ln\left(\frac{T}{S}\right)\right)^2 + e\left(\frac{1}{S^2}\right) + f\left(\frac{\ln\left(\frac{T}{S}\right)}{S}\right) + g\left(\ln\left(\frac{T}{S}\right)\right)^3 + e\left(\frac{1}{S^3}\right) + h\left(\frac{\ln\left(\frac{T}{S}\right)}{S^3}\right) + i\left(\frac{(\ln\left(\frac{T}{S}\right))^2}{S}\right) \quad \mu H/m \quad (4)$$

Where a , b , c , d ,e , f , g , h ,i, j are constant. The constant values are given in Table 5

Table 5 values of constant

Variable	Lower Limit	Upper Limit	Selected constant values
a	0.1109873	0.116483	0.11373491
b	$-4.1745932 \times 10^{-02}$	-0.03820633	$-3.997613 \times 10^{-02}$
c	8.92964×10^{-02}	0.17597573	0.13263606
d	1.479114×10^{-02}	$1.72565629 \times 10^{-02}$	$1.6023852 \times 10^{-02}$
e	-0.5097305	$-8.2211799 \times 10^{-02}$	-0.29597112
f	$-7.4460442 \times 10^{-02}$	$-4.95783943 \times 10^{-02}$	$-6.2019418 \times 10^{-02}$
g	$3.92131547 \times 10^{-03}$	$4.5879355 \times 10^{-03}$	$4.25462548 \times 10^{-03}$
h	$6.493763635 \times 10^{-02}$	0.724256524	0.39459707
i	$8.03465070 \times 10^{-03}$	$5.95222529 \times 10^{-02}$	$3.37784518 \times 10^{-02}$
j	$-8.9953249 \times 10^{-03}$	$-3.9076839 \times 10^{-03}$	$-6.45150442 \times 10^{-03}$

In Table 5 the lower limit and upper limit of each constant value are given. In order to get accurate value of inductance gradient of the rails the constant values are chosen between lower limit and upper limit. The selected suitable constant values are given in the same table.The L' values obtained using empirical formula, for a different rail dimensions, are compared with FEM simulation and given in Table 6. From the table, it is observed that the L' values obtained using regression analysis equation shows a good agreement with value obtained using FEM method.

Table 6 comparison of inductance gradient values with simulation

S.No	Rail dimensions	L' from Ansoft field simulator (μH/m)	Method adopted	Regression analysis Empirical formula (μH/m)
1	T/S =1 , (θ) = 10 ° S = 9 cm	0.1249	Finite Element analysis with higher frequency	0..1253
2	T/S =0.1 , (θ) = 10° S = 3cm	0..2916	do	0.2992

In order to analyze more detail about the equation, the graphs are plotted for various values of T/S and rail separation as shown in Figure 5 and Figure 6.
From the figures it is observed that the value obtained from the simulation and regression analysis equation shows good agreement

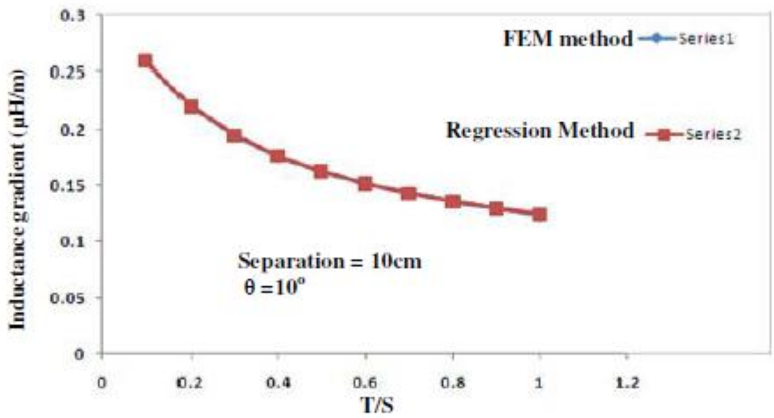

Figure 5 inductance gradient values for various ratio of T/S for rail separation 10cm

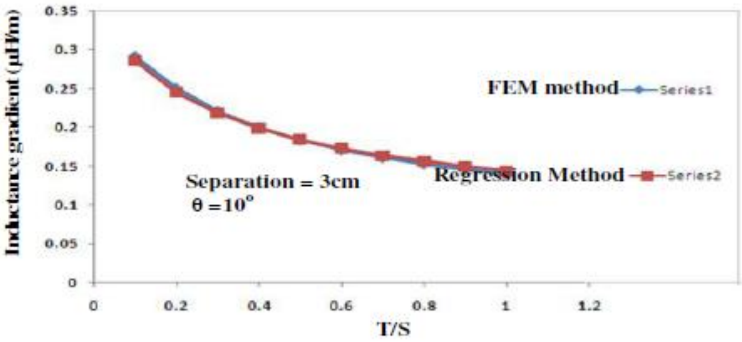

Figure 6 inductance gradient values for various ratio of T/S for rail separation 3cm

3. Conclusions

In a circular bore geometry, for a various values of T/S ,rail separation and opening angle of the rail, L' values are calculated and entered into a curve fitting software in order to extract a suitable empirical formula which could be used to extract L' of the rails, but a suitable solution could not be obtained. So it has been decided to keep any one parameter of a circular bore geometry rail dimensions as a constant value. In this work, the opening angle of rail has been kept at constant value. For various values of T/S and S, L' values are calculated using FEM simulation. These values are entered into curve fitting software and L' formula has been extracted. The L' values obtained using empirical formula and FEM simulation have been compared and shown a good agreement between the results. The empirical formulas obtained in this work could be utilized to optimize the inductance gradient of rails.

4. References

1] Asghar Keshtkar, "Effect of rail dimension on current distribution and inductance gradient", *IEEE Transaction on Magnetics,* Vol. 41, No. 1, pp. 383-386, 2005.

2] Asghar Keshtkar, Sadjad Bayati and Ahmad Keshtkar, "Derivation of a formula for inductance gradient using intelligent estimation method", *IEEE Trans. Magnetics*, Vol. 45, No. 1, pp. 305-308, 2009.

3] F.Jerry Kerris, "Electrical and thermal modeling of rail guns", *IEEE Transaction on Magnetics*, Vol. 20, No. 2, pp. 399-402, 1984.

4] H.T. Honjo, and R.M.Del vecchic, "A program to compute field and Inductance due to rectangular conductor arbitrarily position in space", *IEEE Transaction on Magnetics,* Vol. MAG 22, No. 6, pp. 1532-1535, 1986.

5] M.A. Huerta, and J.C.Nearing, "Conformal mapping calculation of rail gun skin inductance", *IEEE Transaction on Magnetics*, Vol. 27, No. 1, pp. 112-115, 1991.

6] R..Murugan, and K.Udayakumar, . "Effect of rail dimensions on rail gun design parameters", *IEEE INDICON Conference*, pp. 623-626, 2005.

7] ASOFT", Software technical notes, *Ansoft Corporation*, Pittsburgh, PA USA, pp. 772-778, 832, 1984 -2003.

8] Data fit version 9.0.59' Software technical notes, *Oakdale Engineering*, PA 15071, USA, pp.14-21, 1995-2008.

9] http://www.matthewmassey.com.