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HEAT AND MASS TRANSFER ALONG A WEDGE WITH VARIABLE SURFACE TENSION AND HEAT RADIATION IN THE PRESENCE OF SUCTION OR INJECTION

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Abstract

In the present study, an analysis is carried out to study the effects of heat and mass transfer on laminar flow over a wedge with heat radiation in the presence of suction or injection. The governing boundary layer equations are written into a dimensionless form by similarity transformations. The transformed coupled nonlinear ordinary differential equations are solved numerically by using R. K. Gill and

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shooting methods. The effects of different parameters on the dimensionless velocity, temperature, and concentration profiles are shown graphically. Comparisons with previously published works are performed and excellent agreement between the results is obtained.

Introduction

In many transport processes in nature and in industrial applications in which heat and mass transfer is a consequence of buoyancy effects caused by diffusion of heat by chemical species. The study of such processes is useful for improving a number of chemical technologies, such as polymer production and food processing. In nature, the presence of pure air or water is impossible. Some foreign mass may be present either naturally or mixed with the air or water. The present trend in the field of variable viscosity analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular, the study of heat and mass transfer with magnetic effect is of considerable importance in chemical and hydrometallurgical industries.

Many practical diffusive operations involve the molecular diffusion of a species in the presence of heat radiation within or at the boundary. The study of heat and mass transfer with heat radiation is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. The flow of a fluid past a wedge is of fundamental importance since this type of flow constitutes a general and wide class of flows in which the free stream velocity is proportional to a power of the length coordinate measured from the stagnation point.

In these types of problems, the well-known Falkner-Skan transformation is used to reduce boundary-layer equations into ordinary differential equations for similar flows [5]. It can also be used for non-similar flows for convenience in numerical work because it reduces, even if it does not eliminate, dependence on the x-coordinate. The solutions of the Falkner-Skan equations are sometimes referred to as wedge flow solutions with only two of the wedge flows being common in practice [3]. The dimensionless parameter, m plays an important role in such type of problems because it denotes the shape factor of the velocity profiles. It has been shown [7] that when $m < 0$ (increasing pressure), the velocity profiles have point of inflexion whereas when $m > 0$ (decreasing pressure), there is no point of inflexion. This fact is of great importance in the analysis of the stability of laminar flows with a pressure gradient. Yih [9] presented an analysis of the forced convection boundary layer flow over a wedge with uniform suction/blowing, whereas Watanabe [8] investigated the behavior of the boundary layer over a wedge with suction/injection in forced flow. Recently, laminar boundary layer flow over a wedge with suction/injection has been discussed by Kafoussias and Nanousis [6] and Anjali Devi and Kandasamy [1] analyzed the effects of thermal stratification on laminar boundary layer flow over a wedge with suction/injection. Chen [4] has studied the thermal response behavior of laminar boundary layers in wedge flow.

Since no attempt has been made to analyze nonlinear boundary layer flow with heat and mass transfer over a wedge with heat radiation in the presence of suction or injection at the wall of the wedge, we have investigated it in this article. The similarity transformation has been utilized to convert the governing partial differential equations into ordinary differential equations and then the numerical solution of the problem is drawn using R. K. Gill method. Numerical calculations were carried out for different values of dimensionless parameters of the problem under consideration for the purpose of illustrating the results graphically. Examination of such flow models reveals the influence of heat radiation on velocity, temperature and concentration profiles. The analysis of the results obtained shows that the flow field is influenced appreciably by the presence of heat radiation and suction at the wall of the wedge.

Mathematical Analysis

Two-dimensional laminar boundary-layer flow of a viscous and incompressible fluid over a wall of the wedge with suction or injection is analyzed. As shown in Figure 1, x-axis is parallel to the wedge and y-axis is taken normal to it. The fluid properties are assumed to be constant in a limited temperature range. The concentration of diffusing species is very small in comparison to other chemical species, the concentration of

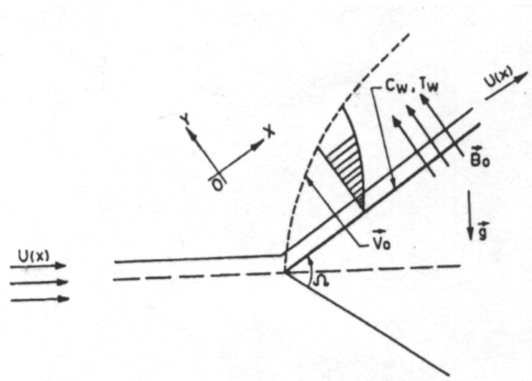

Figure 1. Flow analysis along the wall of the wedge.

species far from the wall, C_∞, is infinitesimally very small [2] and hence the Soret and Dufour effects are neglected. The chemical reactions are not taking place in the flow and the physical properties μ, D and ρ are constant throughout the fluid. Under these conditions, the momentum, energy and diffusion equations for the stated boundary layer for mixed convection flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + U\frac{dU}{dx}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left[\frac{\partial^2 T}{\partial y^2} - \frac{1}{k}\left(\frac{\partial q_r}{\partial y}\right)\right], \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}. \tag{4}$$

The boundary conditions are

$$u = 0,\ v = v_0,\ T = T_w,\ C = C_w \ \text{at}\ y = 0,$$

$$u = U(x),\ T = T_\infty,\ C = C_\infty \ \text{at}\ y \to \infty, \tag{5}$$

where $q_r = -\frac{4\sigma}{3\mu}\frac{\partial T^4}{\partial y}$.

As in [6] we introduce the following change of variables

$$\psi = \sqrt{\frac{2U\nu x}{1+m}} f(x, \eta), \tag{6}$$

$$\eta = y\sqrt{\frac{(1+m)U}{2\nu x}}. \tag{7}$$

Under this consideration, the potential flow velocity can be written [6] as

$$U(x) = Ax^m; \quad \beta = \frac{2m}{1+m}, \tag{8}$$

where A is a constant and β is the Hartree pressure gradient parameter that corresponds to $\beta = \frac{\Omega}{\pi}$ for a total angle Ω of the wedge.

The velocity components are given by

$$u = \frac{\partial \psi}{\partial y}; \quad v = -\frac{\partial \psi}{\partial x}. \tag{9}$$

It can be easily verified that the continuity equation (1) is identically satisfied and introduce the non-dimensional form of temperature and the concentration as

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \tag{10}$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty}, \tag{11}$$

$$\Pr = \frac{\nu}{\alpha} \quad \text{(Prandtl number)}, \tag{12}$$

$$Sc = \frac{\nu}{D} \quad \text{(Schmidt number)}, \tag{13}$$

$$S = -v_0\sqrt{\frac{(1+m)x}{2\nu U}} \quad \text{(suction or injection parameter)}, \tag{14}$$

$$M = \frac{4\sigma T_w^3}{k\mu} \quad \text{(heat radiation parameter)}. \tag{14a}$$

Now the equations (2) to (4) become

$$f''' + ff'' + \frac{2m}{1+m}(1 - f'^2) = 0, \tag{15}$$

$$\theta'' + (1 + \frac{4}{3}M\theta^3) + \Pr f\theta' + 4\theta^2\theta'^2 - \frac{2\Pr}{1+m}f'\theta = 0, \tag{16}$$

$$\phi'' + Sc\ f\theta' - \frac{2Sc}{1+m}f'\phi = 0 \tag{17}$$

with boundary conditions

$$\eta = 0;\ f(0) = \frac{2}{1+m}S,\ f'(0) = 0,\ \theta(0) = 1,\ \phi(0) = 1,$$

$$\eta \to \infty;\ f'(\infty) = 1,\ \theta(\infty) = 0,\ \phi(\infty) = 0, \tag{18}$$

where S is the suction if $S > 0$ and injection if $S < 0$.

Numerical Solution and Discussion

Equations (15) to (17) with boundary conditions (18) were solved numerically using Runge Kutta Gill and shooting methods. The computations have been carried out for various values of heat radiation, M, suction/injection parameter, S and Schmidt number, Sc. In order to validate our method, we have compared steady state results of skin friction, $f''(0)$ for various values of m (Table 1) with those of [8] and [9] and found them in excellent agreement.

Table 1. Comparison of the values of $f''(0)$ with prediction of Watanabe [8] and Yih [9]

M	[9]	[8]	present	Remark
0.0	0.469600	0.46960	0.46960	$Sc = 0$
0.0141	0.504614	0.50461	0.504612	$\Pr = 0.71$
0.0435	0.568978	0.56898	0.568971	$M = 0.0$
0.2	0.802125	0.80213	0.802122	$S = 1.0$
0.3333	0.927653	0.92765	0.927650	

The velocity, temperature and concentration profiles obtained in the dimensionless form are presented in Figures 2-5 for $\Pr = 0.71$ which represents air at temperature $20°C$ and $Sc = 0.62$ which corresponds to water vapour that represents a diffusion chemical species of most common interest in air. The values of Schmidt number, Sc is chosen to be 0.22, 0.62 and .78, heat radiation parameter, M is chosen to be 0.5, 1.0 and 2.0 and the value of suction, S is chosen to be 3.0, 5.0 and 8.0.

Effects due to the Schmidt number with fixed angle of inclination of the wall of the wedge and uniform suction over the velocity, temperature and concentration are shown through Figures 2, 3 and 4.

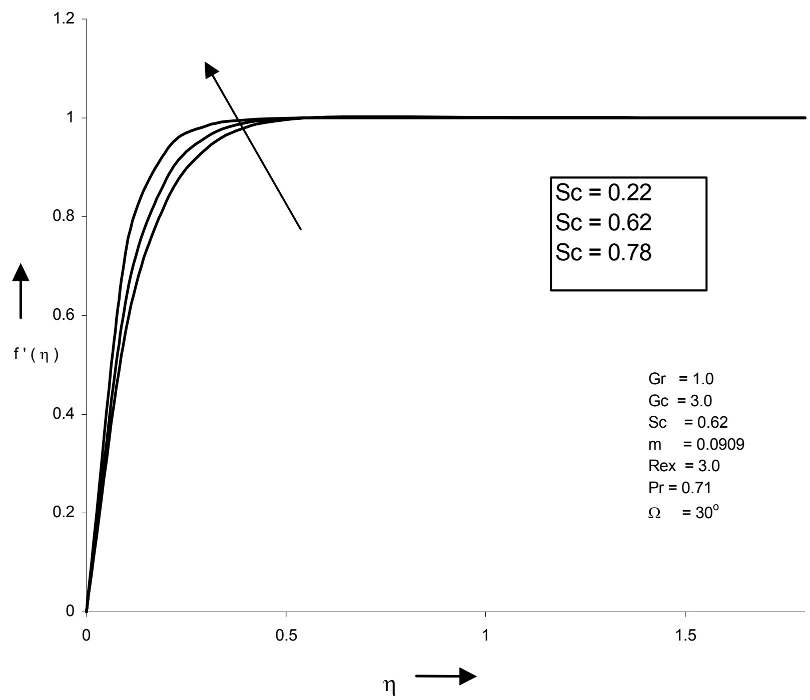

Figure 2. Schmidt number over the velocity profiles with $M = 1.0$, $\Pr = 0.71$, $S = 1.0$ and $m = 0.0909$.

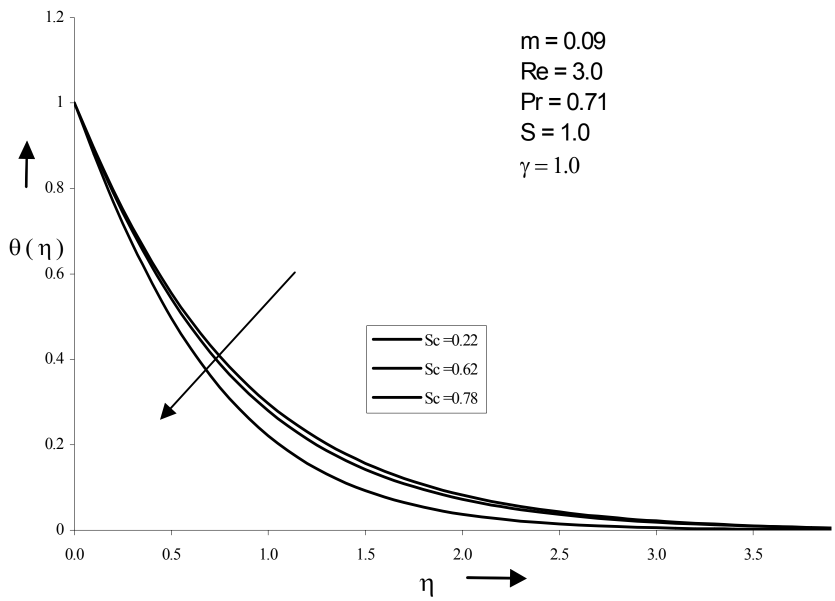

Figure 3. Schmidt number over the temperature profiles with $M = 1.0$, $\mathrm{Pr} = 0.71$, $S = 1.0$ and $m = 0.0909$.

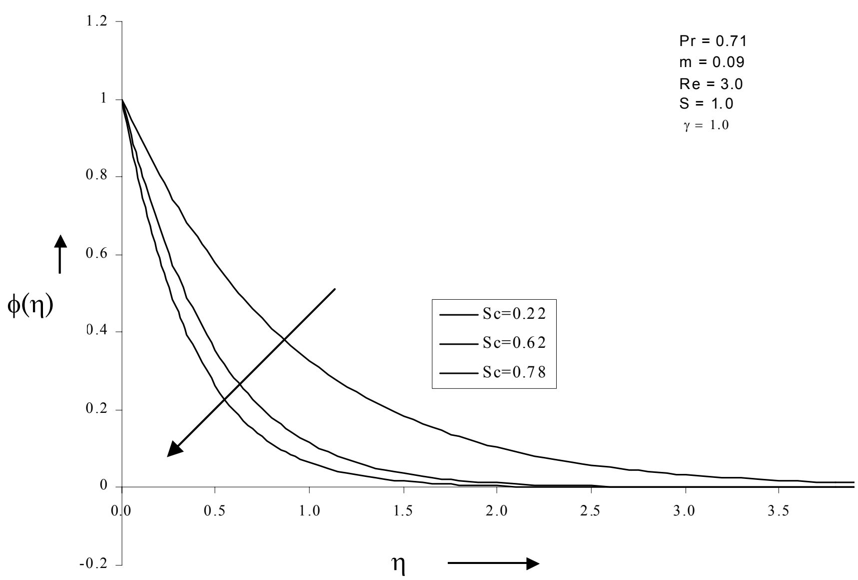

Figure 4. Schmidt number over the concentration profiles with $M = 1.0$, $\mathrm{Pr} = 0.71$, $S = 1.0$ and $m = 0.0909$.

The effect of Prandtl number over velocity, temperature and concentration of the fluid along the wall of the wedge are shown through Figures 5, 6 and 7.

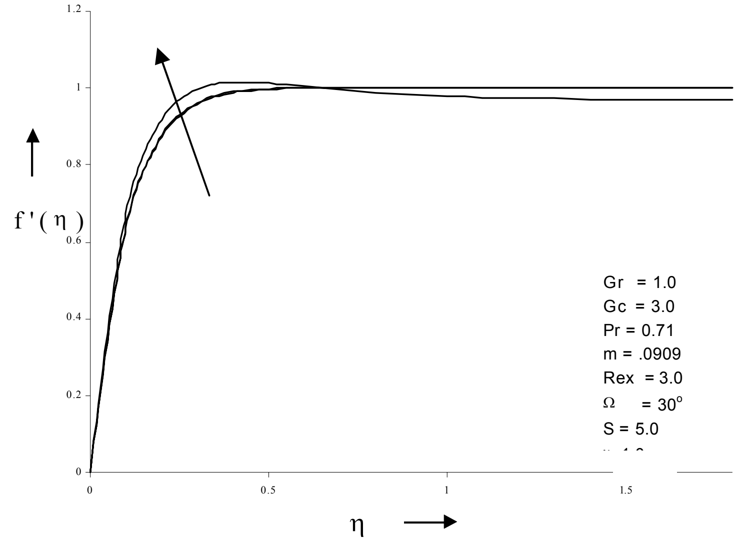

Figure 5. Heat radiation over the velocity profiles with $Sc = 0.62$, $\Pr = 0.71$, $S = 1.0$ and $m = 0.0909$.

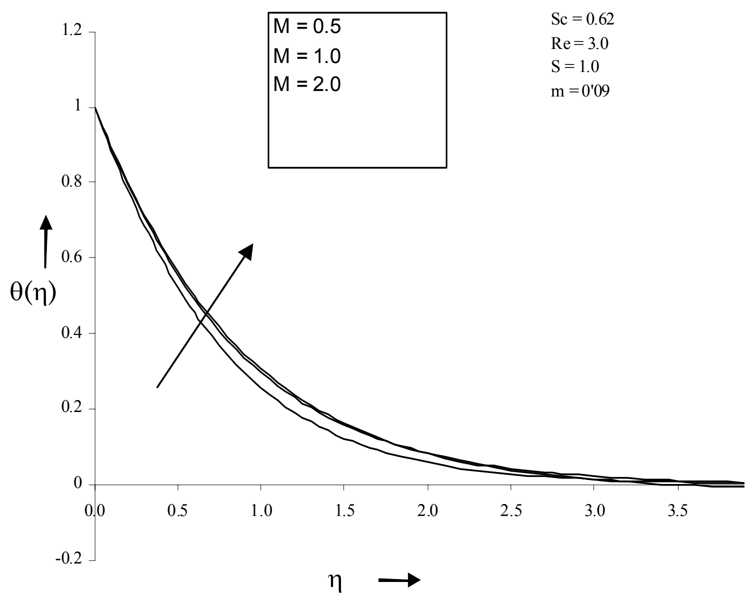

Figure 6. Heat radiation over the temperature profiles with $Sc = 0.62$, $\Pr = 0.71$, $S = 1.0$ and $m = 0.0909$.

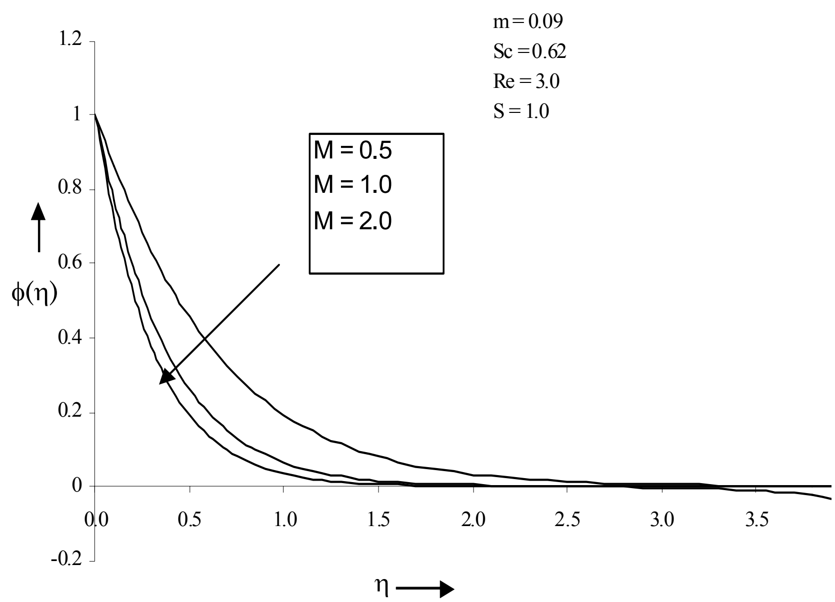

Figure 7. Heat radiation over the concentration profiles with $Sc = 0.62$, $\Pr = 0.71$, $S = 1.0$ and $m = 0.0909$.

Figure 2 depicts the dimensionless velocity profiles $f'(\eta)$ for different values of Schmidt number (*Sc*). Due to the uniform suction with fixed angle of inclination of the wall of the wedge, it is observed that the velocity of the fluid along the wall of the wedge increases with increase of Schmidt number. On the contrary, the dimensionless temperature $\theta(\eta)$ and concentration $\phi(\eta)$ of the fluid reduce with increase of Schmidt number and these are shown in Figures 3 and 4, respectively. All these physical behaviors are due to the combined effects of suction at the wall of the wedge and Schmidt reaction.

Figure 5 represents the dimensionless velocity profiles $f'(\eta)$ for different values of the heat radiation parameter. For uniform suction, it is seen that the velocity and temperature of the fluid increase and the concentration of the fluid decreases with increase of heat radiation along the wall of the wedge and these are shown in Figures 5, 6 and 7, respectively. So, in the case of uniform suction, the increase in heat radiation accelerates the fluid motion and temperature distribution and decelerates the concentration of the fluid along the wall of the wedge.

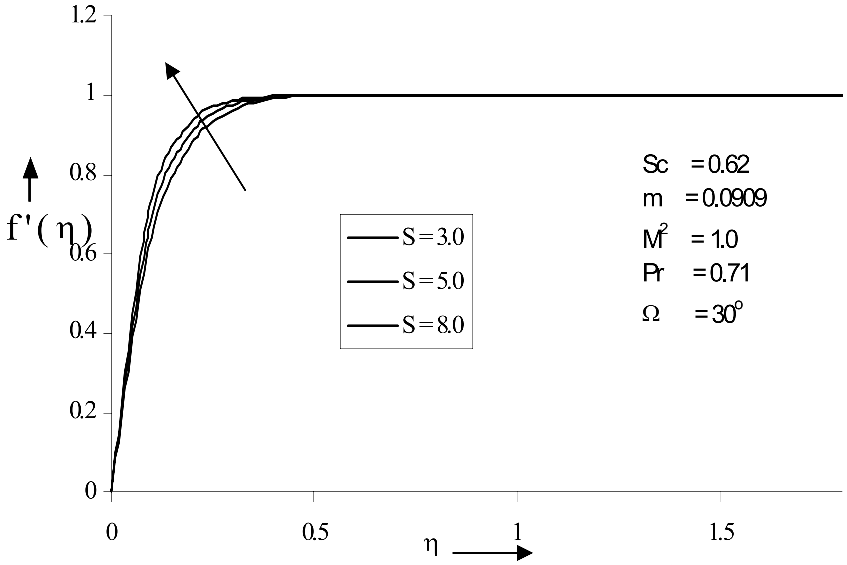

Figure 8. Effects of sunction over the velocity profiles with $M = 1.0$, $\Pr = 0.71$, $Sc = 0.62$ and $m = 0.0909$.

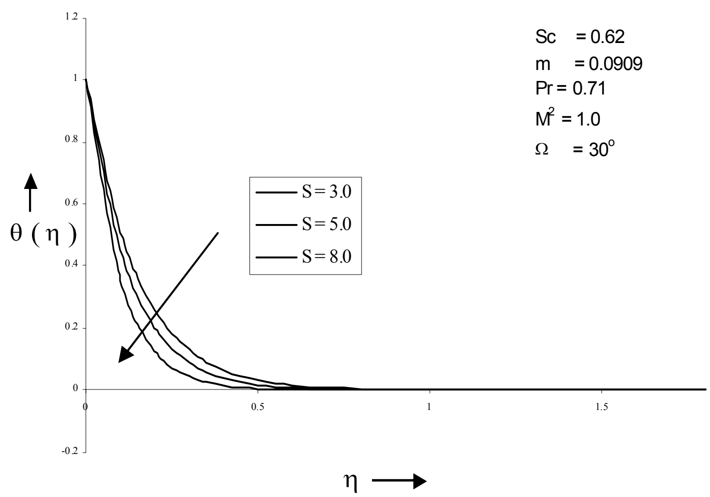

Figure 9. Effects of sunction over the temperature profiles with $M = 1.0$, $\Pr = 0.71$, $Sc = 0.62$ and $m = 0.0909$.

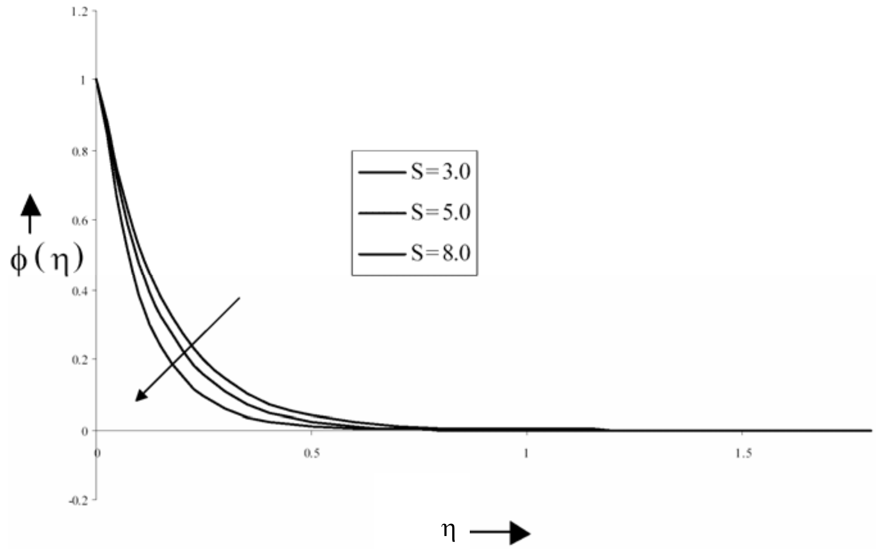

Figure 10. Influence of sunction over the concentration profiles with $M = 1.0$, $\Pr = 0.71$, $Sc = 0.62$ and $m = 0.0909$.

Figure 8 represents the dimensionless velocity profiles for different values of suction parameter $(S > 0)$. In the presence of uniform heat radiation, it is clear that the velocity of the fluid increases and the dimensionless temperature $\theta(\eta)$ and concentration $\phi(\eta)$ of the fluid reduce with increase of suction and these are shown in Figures 8, 9 and 10, respectively.

Conclusion

This paper studied the effects of heat and mass transfer along a wedge with variable surface tension and heat radiation in the presence of suction or injection. The results are presented graphically and the conclusion is drawn that the flow field and other quantities of physical interest are significantly influenced by these parameters. Comparisons with previously published works are performed and excellent agreement between the results is obtained.

We conclude the following from the previous results and discussions:

∗ In the presence of uniform heat source and suction at the wall, it is interesting to note that the velocity of the fluid increases and the temperature and concentration of the fluid decrease with increase of Schmidt number.

∗ Due to the uniform suction with fixed Schmidt number, the increase of heat radiation accelerates the fluid motion and temperature distribution and decelerates the concentration of the fluid along the wall of the wedge.

∗ In the presence of heat radiation, the velocity of the fluid increases and the temperature and concentration of the fluid reduce with increase of suction. All these physical behavior are due to the combined effects of suction and heat radiation along the wall of wedge.

∗ Decrease of the concentration field due to increase in Sc shows that it increases gradually as we replace Hydrogen ($Sc = 0.22$) by water vapour ($Sc = 0.62$) and Ammonia ($Sc = 0.78$) in the said sequence. It is also observed that the effect due to change in Schmidt number is very important in the concentration field.

It is hoped that the present investigation of the study of physics of flow over a wedge can be utilized as the basis for many scientific and engineering applications and for studying more complex vertical problems involving the flow of electrically conducting fluids. The findings may be useful for the study of movement of oil or gas and water through the reservoir of an oil or gas field, in the migration of underground water and in the filtration and water purification processes. The results of the problem are also of great interest in geophysics in the study of interaction of the geomagnetic field with the fluid in the geothermal region.

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