Journal of Applied Nonlinear Dynamics 4(1) (2015) 11–19

Journal of Applied Nonlinear Dynamics

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Numerical Solution of Fuzzy Delay Functional Differential Equations by Euler Method

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Submission Info

Communicated by J.A.T. Machado
Received 6 December 2013
Accepted 27 March 2014
Available online 1 April 2015

Keywords

Fuzzy delay functional -differential equations
Euler method
Fuzzy Cauchy problem

Abstract

In this paper, we study the numerical solution of fuzzy delay functional differential equations by using Euler method. Examples are presented to illustrate the computational aspects of the above method.

1 Introduction

Since many physical problems are modeled by fuzzy differential equations, the numerical solutions of such fuzzy differential equations have been highly studied by many authors. In recent years, numerous works have been focusing on the development of more advanced and efficient methods for numerical solutions of fuzzy differential equations such as Taylor method [1], Runge-Kutta method [2,3], Predictor-corrector method [4]. Subsequently, there have been many papers discussing the numerical solution of hybrid fuzzy differential equations by using different techniques [5–9]. Recently, Khastan et al. [10], proved the existence of solutions of fuzzy delay differential equations under generalized differentiability. However, none of them propose a numerical solutions of fuzzy delay functional differential equations. The topics of differential and integral equations as much as delay functional differential equations has become an important area of investigation in recent years stimulated by their many applications to problems arise in engineering, medicine and biology etc [11]. Prakash et al. in [8] studied the theory of existence of solutions of fuzzy neutral differential equations in Banach spaces by using the Schauder fixed point theorem. Lupulescu [12] established the local and global existence and uniqueness results for fuzzy functional differential equations by using the method of successive approximations and

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ISSN 2164 – 6457, eISSN 2164 – 6473/$-see front materials
DOI : 10.5890/JAND.2015.03.002

contraction principle. Further he studied the fuzzy differential equations with distributed delays and fuzzy population models. The motivation of this paper, we study the numerical solution of fuzzy delay functional differential equations by using Euler method.

2 Preliminaries

Let $P_k(\mathbb{R}^n)$ denote the family of all non-empty, compact, convex subsets of $\mathbb{R}^n$. If α, β $\in \mathbb{R}$ and $A, B \in P_k(\mathbb{R}^n)$ then

$$\alpha(A+B) = \alpha A + \alpha B, \quad \alpha(\beta A) = (\alpha\beta)A, \quad IA = A.$$

and if $\alpha, \beta \geq 0$ then $(\alpha+\beta)A = \alpha A + \beta A$. Denote by E^n the set of $u : \mathbb{R}^n \to [0,1]$ such that

(i) u is normal, that is, there exist $x_0 \in \mathbb{R}^n$ with $u(x_0) = 1$.

(ii) u is convex fuzzy sets that is,

$$u(\lambda x + (1-\lambda)y) \geq \min(u(x), u(y)) \quad \text{for all } x, y \in \mathbb{R}^n \text{ and } \lambda \in [0,1].$$

(iii) u is uppersemicontinuous on $\mathbb{R}^n$.

(iv) $[u]^0 = cl\{x \in R^n : u(x) > 0\}$ is a compact set.

For $0 < \alpha \leq 1$, denote $[u]^\alpha = \{x \in \mathbb{R}^n | u(x) \geq \alpha\}$. Then, from (i)–(iv), it follows that the α - level sets $[u]^\alpha \in P_k(\mathbb{R}^n)$ for all $0 \leq \alpha \leq 1$.
An example of a $u \in E^1$ is given by

$$u(x) = \begin{cases} 4x-3, & \text{if } x \in (0.75, 1], \\ -2x+3, & \text{if } x \in (1, 1.5), \\ 0, & \text{if } x \notin (0.75, 1.5). \end{cases} \tag{1}$$

The α-level sets are given by

$$[u]^\alpha = [0.75 + 0.25\alpha, 1.5 - 0.5\alpha].$$

Let I be a real interval. A mapping $y : I \to E$ is called a fuzzy process and its α-level set is denoted by $[y(t)]^\alpha = [\underline{y}^\alpha(t), \overline{y}^\alpha(t)],\ t \in I,\ \alpha \in (0,1]$.

Triangular fuzzy numbers are those fuzzy sets in E which are characterized by an ordered triple $(x^l, x^c, x^r) \in \mathbb{R}^3$ with $x^l \leq x^c \leq x^r$ such that $[U]^0 = [x^l, x^r]$ and $[U]^1 = \{x^c\}$, then

$$[U]^\alpha = [x^c - (1-\alpha)(x^c - x^l), x^c + (1-\alpha)(x^r - x^c)], \tag{2}$$

for any $\alpha \in [0,1]$.

We shall write E instead of E^1.

Definition 1 The supremum metric d_∞ on E is defined by

$$d_\infty(U,V) = \sup\{d_H([U]^\alpha, [V]^\alpha) : \alpha \in I\},$$

and (E, d_∞) is a complete metric space.

Definition 2 A mapping $F : I \to E$ is Hukuhara differentiable at $t_0 \in I \subseteq \mathbb{R}$ if for some $h_0 > 0$ the

Hukuhara differences $F(t_0+\Delta t) \sim_h F(t_0)$, $F(t_0) \sim_h F(t_0-\Delta t)$, exist in E for all $0 < \Delta t < h_0$ and if there exists an $F'(t_0) \in E$ such that

$$\lim_{\Delta t \to 0^+} d_\infty((F(t_0+\Delta t) \sim_h F(t_0))/\Delta t, F'(t_0)) = 0,$$

and

$$\lim_{\Delta t \to 0^+} d_\infty((F(t_0) \sim_h F(t_0-\Delta t))/\Delta t, F'(t_0)) = 0,$$

the fuzzy set $F'(t_0)$ is called the Hukuhara derivative of F at t_0.

Recall that $U \sim_h V = W \in E$ are defined on level sets, where $[U]^\alpha \sim_h [V]^\alpha = [W]^\alpha$ for all $\alpha \in [0,1]$. By consideration of definition of the metric d_∞, all the level set mappings $[F(.)]^\alpha$ are Hukuhara differentiable at t_0 with Hukuhara derivatives $[F'(t_0)]^\alpha$ for each $\alpha \in [0,1]$ when $F : I \to E$ is Hukuhara differentiable at t_0 with Hukuhara derivative $F'(t_0)$.

Definition 3 A mapping $y : I \to E$ is called a fuzzy process. We denote

$$[y(t)]^\alpha = [\underline{y}^\alpha(t), \overline{y}^\alpha(t)],\ t \in I,\ \alpha \in (0,1].$$

The Seikkala derivative $y'(t)$ of a fuzzy process y is defined by provided the equation defines a fuzzy number $y'(t) \in E$.

We shall use the notation. For $v \in R$ the norm of v is defined by $\|v\| = |v|$. For arbitrary real numbers t and t', $t' \geq t$, the notation $C^p([t,t'])$ denotes the set of all real valued functions on $[t,t']$ with p continuous derivatives. We shall write $C([t,t'])$ instead of $C^0([t,t'])$.
Let I_1 be an interval in $[t,t']$, For $x \in C([t,t'])$ we define $\| x \|^{I_1}$ by

$$\| x \|^{I_1} = \max_{s \in I_1} \|x(s)\|.$$

3 A fuzzy Cauchy problem

Prior to defining a fuzzy Cauchy problem we follow [13] and represent an arbitrary fuzzy number with compact support by a pair of functions $(\underline{u}(\alpha), \overline{u}(\alpha)$, $0 \leq \alpha \leq 1$ which satisfy the following requirement:

1. $\underline{u}(\alpha)$ is a bounded left continuous nondecreasing function over $[0,1]$.
2. $\overline{u}(\alpha)$ is a bounded left continuous nonincreasing function over $[0,1]$.
3. $\underline{u}(\alpha) \leq \overline{u}(\alpha), 0 \leq \alpha \leq 1$

A tilde is placed over a symbol to denote a fuzzy set so $\widetilde{\alpha}, \widetilde{f}(t), \ldots$

This fuzzy number space as shown in [1], can be embedded into the Banach space $B = \overline{C}[0,1] \times \overline{C}[0,1]$ where the metric is usually defined as

$$\| (u,v) \| = \max\{ \sup_{0 \leq \alpha \leq 1} |u(\alpha)|, \sup_{0 \leq \alpha \leq 1} |v(\alpha)| \}, \tag{3}$$

for arbitrary $(u,v) \in \overline{C}[0,1] \times \overline{C}[0,1]$. Throughout this paper we will use the sup norm given by equation (3) rather than the L_p norm. Using this norm enables to obtain stronger results related to the numerical procedure.

Consider the initial value problem for fuzzy delay functional differential equation

$$\begin{aligned} \tilde{y}'(t) &= \tilde{f}(t, y(.)),\ t \in [a,b],\\ \tilde{y}(t) &= \tilde{g}(t),\ t \in [\tau, a], \end{aligned} \tag{4}$$

$\tau \leq a \leq b$, where f is Volterra operator (that is , f depends only on $y(s)$ for $\tau \leq s \leq t$) and g is a prescribed initial function. We can replace the equation (4) by equivalent system

$$\begin{aligned} \underline{y}'(t) &= \underline{f}(t,y(.)) = F(t,\underline{y}(.),\overline{y}(.)), \quad t \in [a,b],\\ \overline{y}'(t) &= \overline{f}(t,y(.)) = G(t,\underline{y}(.),\overline{y}(.)), \quad t \in [a,b],\\ \overline{y}(t) &= \overline{g}(t), \quad t \in [\tau,a],\\ \underline{y}(t) &= \underline{g}(t), \quad t \in [\tau,a], \end{aligned} \tag{5}$$

which possesses a unique solution $(\underline{y},\overline{y})$ which is a fuzzy function, i.e. for each t, the pair $(\underline{y}(t,\alpha),\overline{y}(t,\alpha)$ is a fuzzy number. The parametric form of equation (5) is given by

$$\begin{aligned} \underline{y}'(t,\alpha) &= F(t,\underline{y}(.,\alpha),\overline{y}(.,\alpha)), \quad t \in [a,b]\\ \overline{y}'(t,\alpha) &= G(t,\underline{y}(.,\alpha),\overline{y}(.)), \quad t \in [a,b]\\ \overline{y}(t,\alpha) &= \overline{g}(t,\alpha), t \in [\tau,a]\\ \underline{y}(t,\alpha) &= \underline{g}(t,\alpha), t \in [\tau,a] \end{aligned} \tag{6}$$

for $\alpha \in [0,1]$, the solution of equation (6) must solve equation (5) as well since by the sup norm, an equality between two fuzzy numbers in B yields a pointwise equality.

4 The Euler method

A uniform step h is used, $h = (b-a)/M$ for some integer M, and the grid points are denoted by $t_i = a + ih$ for $i = 0,1,2,\ldots,M$. The sets S and T_h are defined by $S = \{(t,\delta)|, a \leq t \leq b \text{ and } 0 \leq \delta \leq b-t\}$, $T_h = \{t_0,t_1,\ldots,t_{M-1}\}$. For fixed α, $\Delta(y,t,\delta)$ is defined by

$$\Delta(y,t,\delta) = \begin{cases} \frac{1}{\delta}[y(t+\delta,\alpha) - y(t,\alpha)],\ \delta > 0,\\ \qquad\qquad y'(t,\alpha),\ \delta = 0 \end{cases}$$

for all $(t,\delta) \in S$. To compute an approximate solution $\underline{y}_h,\overline{y}_h$ using step h, by Euler method, start by setting $\underline{y}_h(t,\alpha) = \underline{g}_h(t,\alpha)$, $\overline{y}_h(t,\alpha) = \overline{g}_h(t,\alpha)$ for t in the interval $[\tau,a]$, where $\underline{g}_h,\overline{g}_h$ are some continuous approximation to the initial function $\underline{g}(t,\alpha),\overline{g}(t,\alpha)$ respectively. Extend $\underline{y}_h,\overline{y}_h$ continuously to $[\tau,b]$ by induction as follows. Suppose that $\underline{y}_k,\overline{y}_k$ are defined on $[\tau,t]$ for some $t \in T_h$. Compute $F_h(t,\underline{y}_h,\overline{y}_h), G_h(t,\underline{y}_h,\overline{y}_h)$. Extend $\underline{y}_h,\overline{y}_h$ to $[\tau,t+h]$ using the relation

$$\begin{aligned} \underline{y}_h(t+rh,\alpha) &= \underline{y}_h(t,\alpha) + rhF(t,\underline{y}_h,\overline{y}_h), \quad r \in [0,1]\\ \overline{y}_h(t+rh,\alpha) &= \overline{y}_h(t,\alpha) + rhG(t,\underline{y}_h,\overline{y}_h), \quad r \in [0,1] \end{aligned} \tag{7}$$

With this method, after computing and storing $\underline{y}_h(a),\underline{y}_h(a+h),\ldots\underline{y}_h(t)$, $\overline{y}_h(a),\overline{y}_h(a+h),\ldots\overline{y}_h(t)$, $F(a,\underline{y}_h,\overline{y}_h)$, $F_h(a+h,\underline{y}_h,\overline{y}_h),\ldots,F_h(t-h,\underline{y}_h,\overline{y}_h)$ and $G_h(a,\underline{y}_h,\overline{y}_h),G_h(a+h,\underline{y}_h,\overline{y}_h),\ldots,G_h(t-h,\underline{y}_h,\overline{y}_h)$ for $t \in T_h$, if the evaluation of $F_h(t,\underline{y}_h,\overline{y}_h)$ and $G_h(t,\underline{y}_h,\overline{y}_h)$ requires the value of the approximate solution $\underline{y}_h,\overline{y}_h$ at some points other than one of the grid points, say at the point s, then we have either $s < a$, and the approximation $\underline{g}_h,\overline{g}_h$ are used, or $s > a$, relation is used with $t = \max\{t_i < s\}$ and $r = (s-t)/h$. we shall now give some convergence results for general one step methods where the approximate solution $\underline{y}_h$, $\overline{y}_h$ are constructed by replacing (7) by the more general relation

$$\begin{aligned} \underline{y}_h(t+rh,\alpha) &= \underline{y}_h(t,\alpha) + rh\Phi(t,h,\underline{y}_h,\overline{y}_h,r), \quad r\in[0,1], \\ \overline{y}_h(t+rh,\alpha) &= \overline{y}_h(t,\alpha) + rh\Psi(t,h,\overline{y}_h,\overline{y}_h,r), \quad r\in[0,1]. \end{aligned} \tag{8}$$

The functionals $\Phi,\ \psi : S\times C_1[\tau,b]\times C_1[\tau,b]\times[0,1]\to R$ is the increment function of the method. We require $\Phi(t,h,x,y,r)$ and $\Psi(t,h,x,y,r)$to be continuous in r for fixed t,h,x,y and to satisfy the Lipschitz condition.

$$\begin{aligned} \|\Phi(t,h,x_1,y_1,r)-\Phi(t,h,x_2,y_2,r)\| &\le L\max\{\|x_1-x_2\|^{[\tau,t]},\|y_1-y_2\|^{[\tau,t]}\}, \\ \|\Psi(t,h,x_1,y_2,r)-\Psi(t,h,x_2,y_2,r)\| &\le L\max\{\|x_1-x_2\|^{[\tau,t]},\|y_1-y_2\|^{[\tau,t]}\}, \end{aligned} \tag{9}$$

for all $x_1,x_2,y_1,y_2\in C_1[a,b],(t,h)\in S,r\in[0,1]$ where L is constant.

We now give a convergence theorem for Euler method.

Lemma 1. [13] Let a sequence of numbers $\{W_n\}_{n=0}^M$ satisfy

$$|W_{n+1}| \le A|W_n| + B, \quad 0\le n\le M-1 \tag{10}$$

for some given positive constants A and B. Then

$$|W_n| \le A^M|W_0| + B\frac{A^n-1}{A-1}, \quad 0\le n\le M-1. \tag{11}$$

Lemma 2. [13] Let the sequence of numbers $\{W_n\}_{n=0}^M$, $\{V_n\}_{n=0}^M$ satisfy

$$\begin{aligned} |W_{n+1}| &\le |W_n| + A\max\{|W_n|,|V_n|\} + B, \\ |V_{n+1}| &\le |V_n| + A\max\{|V_n|,|W_n|\} + B, \end{aligned} \tag{12}$$

for some given positive constants A and B, and denote

$$U_n = |W_n| + |V_n|, \quad 0\le n\le M.$$

Then

$$U_n \le \overline{A}^n U_0 + \overline{B}\frac{\overline{A}^n-1}{\overline{A}-1}, \quad 1\le n\le M. \tag{13}$$

Lemma 3. If $W\in C([\tau,b])$ and W satisfy the inequality

$$\|W(t)\| \le A\|w\|^{[\tau,t_n]} + B, \quad t\in(t_n,t_{n+1}], \quad 0\le n\le M-1, \tag{14}$$

for some given positive constants A and B. Then

$$\|W(t)\| \le A^N\|W\|^{[\tau,t_n]} + B\frac{A^n-1}{A-1}, \quad 0\le n\le M-1. \tag{15}$$

Proof. By using Lemma 1 we obtained the proof.

Lemma 4. If $W,V\in C([\tau,b])$ and W,V satisfy the inequality

$$\begin{aligned} \|W(t)\| &\le \|W\|^{[\tau,t_n]} + A\max\{\|W\|^{[\tau,t_n]},\|V\|^{[\tau,t_n]}\} + B, \\ \|V(t)\| &\le \|V\|^{[\tau,t_n]} + A\max\{\|W\|^{[\tau,t_n]},\|V\|^{[\tau,t_n]}\} + B, \\ & t\in[t_n,t_{n+1}],\ 0\le n\le M-1 \end{aligned} \tag{16}$$

for some given positive constants A and B, and denote

$$\|U(t)\| = \|W(t)\| + \|V(t)\|, \quad 0 \le n \le M.$$

Then

$$\|U(t)\| \le \overline{A}^n \|U\|^{[\tau,t_n]} + \overline{B}\frac{\overline{A}^n - 1}{\overline{A} - 1}, \quad 1 \le n \le M, \tag{17}$$

where $\overline{A} = 1 + 2A$ and $\overline{B} = 2B$.
Proof. By using Lemma 2, 3 we obtained the proof.

Theorem: Let $\underline{y}_h$ and $\overline{y}_h$ denote the approximate solution computed with increment function Φ and Ψ step h, and initial functions $\underline{g}_h$ and $\overline{g}_h$ If the truncation errors $\Phi(t,h,\underline{y},\overline{y},r) - \Delta(\underline{y},t,rh)$, $\Psi(t,h,\underline{y},\overline{y},r) - \Delta(\overline{y},t,rh)$ satisfy the inequality

$$\|\Phi(t,h,\underline{y},\overline{y},r) - \Delta(\underline{y},t,rh)\| \le E(h,r),$$
$$\|\Psi(t,h,\underline{y},\overline{y},r) - \Delta(\overline{y},t,rh)\| \le E(h,r),$$

for all $t \in T_h$ and all $r \in [0,1]$ then the Euler approximation of (8) converges to exact solutions $\underline{y},\overline{y}$ uniformly on t.
Proof. By (8) with $r = 1$ we have

$$\underline{y}_h(t_i,\alpha) = \underline{g}_h(a,\alpha) + h\sum_{j=0}^{i-1} \phi(t_j,h,\underline{y}_k,\overline{y}_k,1),$$

$$\overline{y}_h(t_i,\alpha) = \overline{g}_h(a,\alpha) + h\sum_{j=0}^{i-1} \psi(t_j,h,\underline{y}_k,\overline{y}_k,1),$$

from the identity $\underline{y}(t+h,\alpha) = \underline{y}(t,\alpha) + h\Delta(\underline{y},t,h)$, and $\overline{y}(t+h,\alpha) = \overline{y}(t,\alpha) + h\Delta(\overline{y},t,h)$we have

$$\underline{y}(t_i,\alpha) = \underline{g}(a,\alpha) + h\sum_{j=0}^{i-1} \Delta(\underline{y},t_j,h),$$

$$\overline{y}(t_i,\alpha) = \overline{g}(a,\alpha) + h\sum_{j=0}^{i-1} \Delta(\overline{y},t_j,h),$$

define the error $W(t,\alpha) = \underline{y}(t,\alpha) - \underline{y}_h(t,\alpha)$, $V(t,\alpha) = \overline{y}(t,\alpha) - \overline{y}_h(t,\alpha)$ From the above two relations and equation(9) we obtained

$$\|W(t_i,\alpha)\| \le \|W(t_i,\alpha)\|^{[\tau,a]} + (b-a)E(h,1) + hL\sum_{j=0}^{i-1} \max\{\|W(t_i,\alpha)\|^{[\tau,a]}, \|V(t_i,\alpha)\|^{[\tau,a]}\},$$

$$\|V(t_i,\alpha)\| \le \|V(t_i,\alpha)\|^{[\tau,a]} + (b-a)E(h,1) + hL\sum_{j=0}^{i-1} \max\{\|W(t_i,\alpha)\|^{[\tau,a]}, \|V(t_i,\alpha)\|^{[\tau,a]}\}.$$

On the other hand, subtracting (8) from the identity,

$$\underline{y}(t+rh,\alpha) = \underline{y}(t,\alpha) + rh\Delta(\underline{y},t,rh),$$
$$\overline{y}(t+rh,\alpha) = \overline{y}(t,\alpha) + rh\Delta(\overline{y},t,rh),$$

we have

$$\|W(t_i+rh,\alpha)\| \leq \|W(t_i,\alpha)\|^{[\tau,t_i]} + hE^*(h) + hL\max\{\|W(\alpha)\|^{[\tau,t_i]}, \|V(\alpha)\|^{[\tau,t_i]}\},$$
$$\|V(t_i+rh,\alpha)\| \leq \|V(t_i,\alpha)\|^{[\tau,t_i]} + hE^*(h) + hL\max\{\|W(\alpha)\|^{[\tau,t_i]}, \|V(\alpha)\|^{[\tau,t_i]}\}.$$

By Lemma 3 and Lemma 4

$$\|W(t)\| \leq (1+2Lh)^n \|U\|^{[\tau,a]} + hE^*(h)\frac{(1+2Lh)^n - 1}{2Lh},$$
$$\|V(t)\| \leq (1+2Lh)^n \|U\|^{[\tau,a[} + hE^*(h)\frac{(1+2Lh)^n - 1}{2Lh},$$

where $\|U\|^{[\tau,a]} = \|W\|^{[\tau,a]} + \|V\|^{[\tau,a]}$, In particular

$$\|W(t)\| \leq (1+2Lh)^n \|U\|^{[\tau,a]} + hE^*(h)\frac{(1+2Lh)^{(b-a)/h} - 1}{2Lh},$$
$$\|V(t)\| \leq (1+2Lh)^n \|U\|^{[\tau,a]} + hE^*(h)\frac{(1+2Lh)^{(b-a)/h} - 1}{2Lh},$$

Since $\|W\|^{[\tau,a]} = \|V\|^{[\tau,a]} = 0$ we obtain

$$\begin{aligned} \|W(t)| &\leq E^*(h)\frac{e^{2L(b-a)} - 1}{2L}h, \\ \|V(t)\| &\leq E^*(h)\frac{e^{2L(b-a)} - 1}{2L}h, \end{aligned} \tag{18}$$

and if $h \to 0$ we get $\|W(t)\| \to 0, \|V(t)\| \to 0$ which concludes the proof.
Triangular fuzzy numbers are shown by

$$\underline{y}_h(t,\alpha) = y_h^c(t,\alpha) - (1-\alpha)(y_h^c(t,\alpha) - y_h^l(t,\alpha)),$$
$$\underline{y}_h(t,\alpha) = y_h^c(t,\alpha) - (1-\alpha)(y_h^r(t,\alpha) - y_h^c(t,\alpha)),$$
$$\underline{y}(t,\alpha) = y^c(t,\alpha) - (1-\alpha)(y^c(t,\alpha) - y^l(t,\alpha)),$$
$$\underline{y}(t,\alpha) = y^c(t,\alpha) - (1-\alpha)(y^r(t,\alpha) - y^c(t,\alpha)).$$

5 Examples

Example 1. Consider linear delay-differential equation of the form

$$\tilde{y}'(t) = \tilde{y}(t) - \tilde{y}(t-1), \quad t \in [0,2]$$

with

$$\tilde{y}(t) = [t + 0.96e^{-t}, t + e^t, t + 1.01e^t], \quad t \in [-1,0].$$

The exact solution:
For $t \in [0,1]$

$$y(t) = \left\{ \begin{array}{c} 0.96e^t + t + 0.505e^{1-t} - 0.505e^{t+1} \\ e^t + t + 0.5e^{1-t} - 0.5e^{t+1} \\ 1.01e^t + t + 0.48e^{1-t} - 0.48e^{t+1} \end{array} \right\}.$$

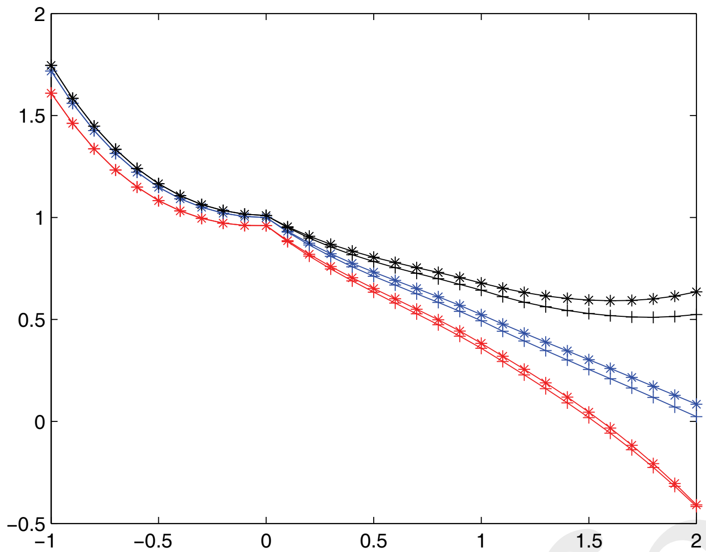

Fig. 1 Comparison of exact and approximation solution. *- y^l, *- y^c, *-y^r, *- y^l_h, +- y^c_h, +-y^r_h.

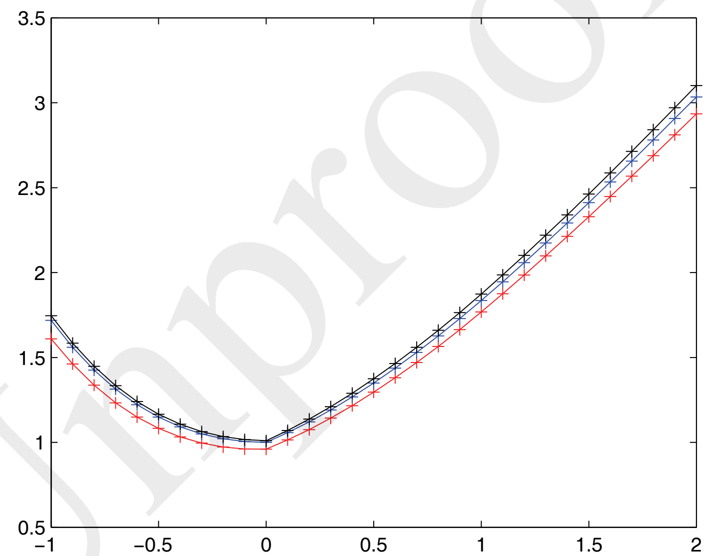

Fig. 2 Approximate solution obtained by Euler method: +- y^l_h, +- y^c_h, +-y^r_h.

For $t \in [1,2]$

$$y(t) = \left\{ \begin{array}{c} 0.24e^t + 1.515e^{t-1} - 0.505e^{t+1} - 1.01te^{t-1} + t + 0.24e^{2-t} + 0.48te^t \\ 0.25e^t + 1.5e^{t-1} - 0.5e^{t+1} - te^{t-1} + t + 0.25e^{2-t} + 0.5te^t \\ 0.2525e^t + 1.44e^{t-1} - 0.48e^{t+1} - 0.96te^{t-1} + t + 0.2525e^{2-t} + 0.505te^t \end{array} \right\}.$$

Comparison of exact and approximate solution obtained by Euler method is shown in Figure 1.

Example 2. Consider nonlinear delay-differential equation with constant delays of the form

$$\tilde{y}'(t) = \sqrt{\tilde{y}(t)} - 0.25\tilde{y}(t-1), \quad t \in [0,2]$$

with

$$\tilde{y}(t) = [t + 0.96e^{-t}, t + e^{t}, t + 1.01e^{t}], \quad t \in [-1,0].$$

Approximate solution obtained by Euler method is shown in Fig. 2.

6 Conclusion

We developed and illustrated numerical algorithms for solving the fuzzy delay functional differential equation by Euler method. The initial value problem of fuzzy delay functional differential equation is replaced by two parametric ordinary differential equations which are then solved numerically using developed algorithm. In this work Euler approximation method is illustrated by solving linear delay differential equation and it is compared with exact solution.

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