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### Thermosolutal combined convection in a lid-driven enclosure with time periodic heating and linearly salting

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Abstract. Thermosoutal combined convective flow with mass and energy transport in a liddriven enclosure box with discrete time periodic heating and linearly varying mass transfer is numerically investigated. The segment of the left wall is heating periodically in time and the right wall is preserved at a lower temperature. The top and bottom walls are thermally insulated. Two different concentrations are imposed along the left and right walls. The mathematical systems are solved by finite volume method. The results are obtained for various values of factors, like, Richardson number, period and amplitude. It is observed that the oscillating frequency of averaged Nusselt number diminishes with raise in the period in the forced and free convection modes. The averaged Sherwood number rises as amplitude raise in the free convection mode. It is also attained that both averaged Sherwood and Nusselt numbers are diminished on raising the Richardson number.

Keywords: Combined convection, double diffusion, time periodic boundary, lid-driven cavity.

#### **1. Introduction**

Combined convection is the results of contact between forced convection and natural convection and it has possible applications in numerous engineering, and technological processes [1-7]. The oscillationinduced heat transport with time periodic boundary conditions has been focused by a number of scholars due to its applications. Abourida et al. [8] numerically evaluated transitory natural convection in a box with horizontal walls with periodic temperatures. Bae et al. [9] numerically deliberated buoyant convective stream in a box with a baffle under periodic wall temperature. Karimipour et al. [10] numerically evaluated the periodic combined convective flow of a nanoliquid in a box with top-lid sinusoidal motion. Lakhal et al. [11] obtained from the transitory natural convective flow in a box partially heated from side that the average Nusselt number and stream strength are significantly different with those found in stationary region.

Saleh et al. [12] found from the numerical study on the influence of time periodic circumstances on convective streams in a porous box with non-uniform interior heating. Sankar et al. [13] carried out a numerical study on convective stream in a porous box with partly thermally active walls. Sivakumar et al. [14] numerically analyzed the influence of partial cooling/heating and interior energy generation on convective stream in a lid-driven enclosure. Sivakumar et al. [15] numerically explored the influence of heater position and length on combined convection in lid-driven enclosures. Sivasankaran and Bhuvaneswari [16] numerically deliberated the influence of thermal zones and external magnetic field on magnetic convection in a box.

In most of the works in the literature, combined convection, mass and energy transports in square or rectangular enclosed spaces are described. Few studies are executed on dual diffusive convection with time periodic heating on sidewall(s) in cavities. So, the current study is focused on mixed thermosolutal convection in a square lid-driven box with time periodic heating along sidewall and linearly varying salting.

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#### 2. Mathematical model



Figure 1. Physical model

The thermosolutal combined convection in a 2-dimensional four-sided enclosure of width *L* is displayed in figure 1. The middle portion of the left wall is heated periodically whereas the opposite sidewall is kept at lower temperature  $T_c$ . The constant length of the heater is considered to be L/2. However, the right wall and left wall are at a higher  $(c_h)$  and lower  $(c_l)$  concentrations with  $c_h > c_l$ . The cavity is filled with air. The flow is regarded as laminar, unsteady, and incompressible. The top wall of the enclosure is permitted to move in its own level at a uniform velocity  $U_0$ . The bottom and top walls of the enclosure are adiabatic and resist to mass transfer. It is presumed that the viscous dissipation is ignored. The fluid thermo-physical properties are supposed to be constant, except the density which is linearly varying with concentration and temperature as  $\rho = \rho_0 [1 - \beta_c (c - c_0) - \beta_T (\theta - \theta_0)]$ , where subscript 0 represents the reference state. As per the above assumptions, the governing model are as follows:

$$u_x + v_y = 0 \tag{1}$$

$$u_{t} + uu_{x} + vu_{y} = -\frac{1}{\rho} p_{x} + v \left( u_{xx} + u_{yy} \right)$$
(2)

$$v_{t} + uv_{x} + vv_{y} = -\frac{1}{\rho} p_{y} + v (v_{xx} + v_{yy}) + g\beta_{T} (T - T_{c}) + g\beta_{c} (c - c_{l})$$
(3)

$$T_t + uT_x + vT_y = \alpha \left( T_{xx} + T_{yy} \right)$$
(4)

$$c_t + u c_x + v c_y = D \left( c_{xx} + c_{yy} \right)$$
(5)

The initial and boundary settings are:

$$u=0, v=0, T=T_{c}, c=c_{l}, 0 \le x \le L, 0 \le y \le L$$
  

$$u=0, v=0; \quad T_{y}=0, c_{y}=0 \qquad y=0$$
  

$$u=U_{0}, v=0; \ T_{y}=0, c_{y}=0 \qquad y=L \qquad (6)$$
  

$$u=0, v=0; \ c=c_{l}, \ T=T_{h}=1-a \sin\left(\frac{\pi f t y}{L}\right), \ \frac{L}{4} \le y \le \frac{3L}{4}, \ x=0$$
  

$$\theta_{x}=0, \ 0 \le y \le \frac{L}{4} \text{ and } \frac{3L}{4} \le y \le L, \ x=0$$
  

$$u=v=0; \ T=T_{c}, \ c=c_{h}=1-y, \ x=L$$

Using the following dimensionless variables

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$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{U_0}, \quad V = \frac{v}{U_0}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad \tau = \frac{tU_0}{L}, \quad \zeta = \omega L/U_0, \quad \Psi = \psi/LU_0$$

$$C = \frac{c - c_l}{c_h - c_l}, \ f = \frac{1}{p}, \ P = \frac{pU_0}{L} \text{ and } A = (a/T_h)/T_c,$$
(7)

the above model is modified in the vorticity-stream function formulation as follows:

$$\zeta_{\tau} + U\zeta_{X} + V\zeta_{Y} = \frac{1}{\operatorname{Re}} \left( \zeta_{XX} + \zeta_{YY} \right) + Ri \left( \theta_{X} + NC_{X} \right)$$
(8)

$$\Psi_{XX} + \Psi_{YY} = -\zeta \tag{9}$$

$$\theta_{r} + U\theta_{X} + V\theta_{Y} = \frac{1}{\Pr \operatorname{Re}} \left( \theta_{XX} + \theta_{YY} \right)$$
(10)

$$C_{\tau} + UC_{X} + VC_{Y} = \frac{1}{Sc \operatorname{Re}} \Big( C_{XX} + C_{YY} \Big)$$
(11)

where  $U = \Psi_Y$ ,  $V = -\Psi_X$  and  $\zeta = V_X - U_Y$ 

The dimensionless constraints are defined as follows:  $Gr_T = (g\beta_T (\theta_h - \theta_c)L^3)/v^2$  is thermal Grashof number,  $Gr_C = (g\beta_c (c_h - c_l)L^3)/v^2$  is solutal Grashof number,  $N = \frac{\beta_c (c_h - c_l)}{\beta_T (\theta_h - \theta_c)} = \frac{Gr_C}{Gr_T}$  is buoyancy ratio,  $Pr = v/\alpha$  is Prandtl number,  $Re = U_0 L/v$  is Reynolds number,  $Ri = \frac{Gr_T}{Re^2}$  is Richardson number and  $Sc = \frac{v}{D}$ is Schmidt number. The dimensionless initial and boundary situations area

is Schmidt number. The dimensionless initial and boundary situations are:

For 
$$\tau = 0$$
:  $U = 0; V = 0, \quad \theta = 0, C = 0$   $0 \le (X, Y) \le 1,$   
For  $\tau > 0$ :  $U = 0; V = 0, \quad \theta_Y = 0, \quad C_Y = 0, \quad Y = 0$   
 $U = 1; V = 0, \quad \theta_Y = 0, \quad C_Y = 0, \quad Y = 1$   
 $U = 0; V = 0, \quad C = 0, \quad \theta = \theta_h = 1 - A \sin\left(\frac{\pi \tau Y}{P}\right), \quad \frac{1}{4} \le X \le \frac{3}{4}$   
 $\theta_X = 0, \quad 0 \le Y \le \frac{1}{4} \text{ and } \frac{3}{4} \le Y \le 1, \quad X = 0$   
 $U = 0; V = 0, \quad \theta = 0, \quad C = 1 - Y, \quad X = L$ 

The local heat transfer and the average heat transfer rate at the heater on left wall are calculated by  $Nu = -(\theta_x)_{x=0}$  and  $\overline{Nu} = \int_h Nu_l dY$  where h = L/2 is the heater length. The local and average mass

transfer rate at the right wall are expressed by  $Sh = -(C_X)_{X=1}$  and  $\overline{Sh} = \int_0^1 Sh \, dY$ , respectively. The

governing systems are solved by control volume method. The complete process of solution is found in ref. [1-7].

#### 3. Results and discussion

The Period (P) and Amplitude (A) are varied from 1 to 4 and 0.2 to 0.8, respectively. The Richardson number (Ri) is varied from 0.1 to 100. The values of Reynolds number varied in the range of  $\text{Re}=10^{1}$  to 10<sup>3</sup>. The Prandtl number Pr=0.71, Schmidt number  $S_{C}=5$ , Grashof number  $G_{r}=10^{4}$  are fixed. The buoyancy ratio (N) is fixed at N=1.

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Figure 2. Streamlines, Isotherms, Concentrations for different Ri with P = 1 and A = 0.2.



Figure 3. Streamlines, Isotherms, Concentrations for different Ri, A = 0.8 with P = 2.

Figure 2 shows that the stream pattern, isotherms and concentrations reported for diverse values of Ri such as Ri = 0.01, 1 and 100 with fixed value of P = 1 and A = 0.2. Stream is regarded as by a single circulating vortex inside the box for P = 1 in the forced convection mode. The change of Ri from forced convective regime to mixed convective mode, the stream is slightly changed in the enclosure. The middle region of the rotating eddy inside the box move about upward where the top wall is moving from left to right direction. In the case of free convection, there come out a reasonable change in the flow behaviour. The centre region of circulate eddy go to the centre of the enclosure and revolving eddy give

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the sense almost in the entire enclosure. These remarks illustrate that the fluid flow depends only on Ri. The heater is keeping at central point of the left sidewall. When  $R_i = 0.01$ , the temperature distribution occurs beside the middle part of the left side wall. But, no significant temperature distribution is exposed at middle of the enclosure and bottom portion of the left-wall. Further, the heat energy distribution is not effective at bottom of the box. Strong and thin boundary layers are formed along the heating and cooling region. On raising the Ri from 0.01 to 1 the isotherms are dispersed in the entire box and strong thermal boundary layers disappear along the heating region. The same result has been obtained for raising the Richardson number from  $R_i = 1$  to 100. In general; raising Richardson number forms better heat transfer along middle of the left sidewall. The solutal boundary layer in the each wall of the cavity demonstrates that mass distribution is identified very sharp gradients in the forced convection regime. An oval-shaped cell is observed in the centre of the box. When  $R_i = 1$ , the concentration gradients increased and the elliptical cell is deformed due solutal buoyancy force. At  $R_i = 100$ , the concentrations show that the mass distribution take place along the side walls. Moreover, solutal boundary layers are more effective along the right sidewall where the high concentration is located.







Figure 5. Local Sherwood number with Ri=1, N = 1.

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Figure 6. Averaged Nusselt number for various values of Ri, P, A with N = 1.

To analyse the effects of Richardson number, the amplitude and period are set as constant at A = 0.8 and P = 2. Figure 3 illustrates streamlines, isotherms and concentrations for Richardson number

takes the values 0.01 to 100. The appearance of flow field as a single major eddy resides in the whole enclosure in the forced convection regime. The flow formation is to some extent changed in the mixed convection mode. But, considerable change found on raising the Richardson number in the free convection mode. The central region is moving in the middle of the box. When  $R_i = 0.01$ , the heat distribution on the left sidewall is increased whereas the right sidewall is kept fixed. The isotherms show very similar for  $R_i = 1$  and 100. Hence, isotherms are more effect along middle of the left sidewall and make little effect along the right sidewall. The appearance of the solutal layers at boundary in the box demonstrates that species distribution is increased more effectively along the left and right sidewalls. Concentrations occur along both left and right sidewall in the forced convection mode. On raising the Ri to 1 and 100, the solutal boundary layers are diminished on the right sidewall where the high concentration gradient is kept fixed. In general, the solutal boundary layers become noteworthy in the regime of free convection for the period P = 2 and amplitude A = 0.8.

The effects of the period and amplitude on the local heat transfer beside the Y coordinate of the left sidewall for A = 0.2 and N = 1 with Ri = 1 is reported in the figure 4(a). When the change of period, the energy transport in the left-wall has considerable change in the mixed convection mode. It appears that the local energy transport rate rises along with P = 1 to P = 4 whereas it is diminished for P = 3. When the period is P = 4, the local heat transfer is over again increased. It is obtained that the maximum value of local heat transfer is observed at P = 4. The local Nusselt number diminishes according to raise in the amplitude from A = 0.2 to A = 0.8. It is obtained that the amplitude causes raise in the local heat transfer rate. These have been depicted in the figure 4(b). While comparing the local Nusselt number graphs, higher heat transfer is observed for the period change at P = 4. Figure 5(a) and (b) reveals the effect of the period and amplitude on the local mass transfer rate in the mixed convection mode along the right sidewall for the fixed value of P = 2 and N = 1. When Ri = 1, a slender variation is obtained on the local Sherwood number. It is also found with the intention of the local Sherwood number is decreased on lower part of the right sidewall on raising both period and amplitude values from P = 1 to P = 4 and A = 0.2 to A = 0.8.

Figure 6(a)-(c), denotes the variation of averaged Nusselt number of the hot wall temperature for different time and period with different Richardson number for P = 2 and N = 1. When Ri = 0.01, the oscillating frequency diminishes with increase in the period from P = 1 to P = 4. The maximum energy transport rate is attained for P = 1. In the mixed convection mode, the oscillation frequency of energy transport rate is diminished on growing the period upto P = 3 and then meet the same heat transfer rate for P = 4 as like P = 1. The average heat transfer rate is very minimum in the free convection dominated mode. It also indicates that increase in Richardson number; decrease the average Nusselt number whereas it is raised for raising the period. To demonstrate the result of amplitude on the average Nusselt number through the heater, we present in figure 6(d)-(f) for different time and Richardson number with P = 2 and N = 1. Perfectly sinusoidal law curves are found for all amplitude (A = 0.2, 0.4, 0.6 and 0.8). The time dependent average Nusselt number displays oscillatory recital which is always symmetric about the amplitude values. On the other hand, quantitative variations are appeared; the erratic average Nusselt number happens to significantly larger as A raises. Moreover, the overall energy transport rate is decreased on raising the Richardson number. The maximum value of averaged Nusselt number is attained for the forced convective regime.

The averaged Sherwood number along the right sidewall is depicted in the figure 7(a)-(d) for the various period and time with different Richardson number. In the mixed convection dominated mode, it seems that the mass transfer rate decreases with slight variation for increasing time and periods (See figure 7(a)). Perfectly sinusoidal shape curve of average Sherwood number exhibits for increasing the period in the free convection dominated mode. The average Sherwood number lies between -1.74 and - 2.6 for all values of the period. The fluctuating curve of mass transport rate is diminished by increasing the period from P = 1 to 4. It is depicted in figure 7(b). Hence the average Sherwood number is strongly affected in the free convective dominated regime. To find the effect of amplitude on the average Sherwood number with various time and Richardson number for N = 1 and P = 2 are presented in the figure 7(c) and 7(d). When Ri = 1, the overall mass transport rate decreases when the time is increased.

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The considerable change observed on the average Sherwood number for raising the amplitude values. These are demonstrated in figure 7(c). The curves are perfectly in the sinusoidal form for the first two amplitude values (A = 0.2, 0.4). There exist quite a lot of extra twist in the curves of average Sherwood number for A = 0.6 and 0.8. The average mass transfer rate rises as amplitude rise in the free convective mode as shown in figure 7(d).

#### 4. Conclusions

The paper presents a numerical investigation on thermosolutal combined convective stream with energy and mass transfer in a lid-driven box with discrete time periodic heating and linearly varying mass transfer. The stream is independent of the period and amplitude; it considerably changes on changing the Ri. The energy transfer enhances along the middle of the left sidewall due to rise in Ri. The solutal boundary layer rises along the right sidewall on growing Richardson number. The oscillating frequency of averaged Nusselt number declines with raise in the period from P = 1 to P = 4 in the forced and free convection modes. The averaged Sherwood number rises as amplitude rise in the free convection mode. Both averaged Nusselt and Sherwood numbers decline on growing Ri.



Figure 7. Averaged Sherwood number for various values of Ri, P and A with N = 1.

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