Far East Journal of Applied Mathematics
Volume 29, Number 3, December, 2007, Pages 433-444
Published online: November 6, 2007
This paper is available online at http://www.pphmj.com

COMBINED HEAT AND MASS TRANSFER IN MHD FREE CONVECTION FROM A VERTICAL SURFACE WITH OHMIC HEATING IN THE PRESENCE OF SUCTION OR INJECTION

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Abstract

The problem of combined heat and mass transfer of an electrically conducting fluid in MHD natural convection adjacent to a vertical surface is analyzed, taking into account the effects of Ohmic heating in the presence of suction or injection. An approximate numerical solution

2000 Mathematics Subject Classification: 80xx, 74A15.

Keywords and phrases: combined heat and mass transfer, MHD natural convection, Ohmic heating, buoyancy ratio, suction or injection at wall surface.

Communicated by T. S. Zhao

Received February 2, 2007

for the steady laminar boundary-layer flow over a wall of the surface in the presence of species concentration and mass diffusion has been obtained by solving the governing equations using numerical technique. The fluid is assumed to be viscous and incompressible. Numerical calculations are carried out for different values of dimensionless parameters and an analysis of the results obtained shows that the flow field is influenced appreciably by the magnetic effect, the buoyancy ratio between species and thermal diffusion and suction or injection at wall surface. Effects of these major parameters on the transport behaviors are investigated methodically and typical results are illustrated to reveal the tendency of the solutions. Representative results are presented for the velocity, temperature and concentration distributions, as well as the local skin-friction coefficient and skin-friction.

1. Introduction

Simultaneous heat and mass transfer from different geometrics embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors and underground energy transport. Early studies that considered combined heat and mass transfer in natural convection boundary layer flows over heated surface with various geometrics can be found in the monograph by Gebhart et al. [4]. Recently some researchers have been carried out to include various physical aspects of the problem of combined heat and mass transfer. The transient free convection flow has been investigated [8] for an impulsively started vertical plate with heat and mass transfer. Chamkha and Khaled [2] investigated the problem of coupled heat and mass transfer by MHD free convection from an inclined plate in the presence of internal heat generation or absorption. Kandasamy [7] has investigated the problem of the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow with thermal diffusion and thermal stratification in the presence of suction. Combined heat and mass transfer in MHD free convection from a vertical surface with Ohmic heating and viscous dissipation has been discussed [3]. For the problem of coupled heat and mass transfer in MHD free convection, the effects of Ohmic heating in the presence of suction are not studied in the above investigation. However, it is more realistic to include these effects to explore the impact of the magnetic field on the thermal transport in the buoyancy layer. With this awareness, the effect of Ohmic heating on the MHD free convection heat transfer has been examined for a Newtonian fluid [6] and for a micropolar fluid [5]. These effects on combined heat and mass transfer with Ohmic heating in MHD free convection flow have not yet been studied. This study is therefore initiated to investigate the problem of MHD natural convection flow over a vertical surface, taking into consideration the effect of the strength of the magnetic field and Ohmic heating.

Since no attempt has been made to analyze the combined heat and mass transfer in MHD free convection from a vertical surface with Ohmic heating in the presence of suction or injection, we have investigated it in this article. The similarity transformation has been utilized to convert the governing partial differential equations into ordinary differential equations and then the numerical solution of the problem is drawn using Runge-Kutta and Shooting methods. Numerical calculations were carried out for different values of dimensionless parameters of the problem under consideration for the purpose of illustrating the results graphically. Examination of such flow models reveals the influence of the strength of the magnetic field and buoyancy ratio on velocity, temperature and concentration profiles. The analysis of the results obtained shows that the flow field is influenced appreciably by the presence of magnetic effect and buoyancy ratio in the presence of suction or injection at wall surface.

2. Mathematical Analysis

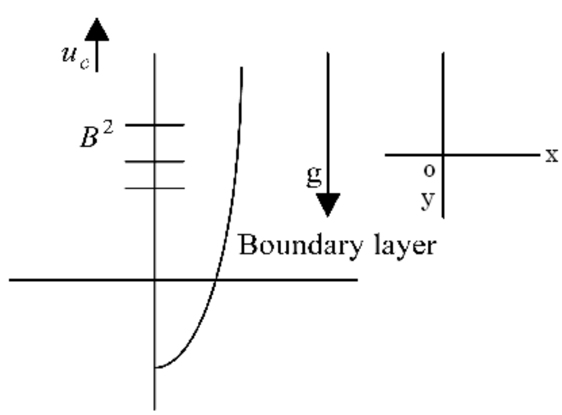

Figure 1. Flow analysis-vertical surface.

Two-dimensional laminar boundary-layer flow of a viscous and incompressible fluid over a vertical surface with suction or injection is analyzed. As shown in Figure 1, the x-axis is parallel to the surface and y-axis is taken normal to it. With x measured along the plate, a magnetic field of strength B is applied in the y-direction that is normal to the streamwise direction. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is negligible in comparison to the applied magnetic field. We further assume that the impressed electric field is zero and the Hall effect is neglected. The fluid properties are assumed to be constant in a limited temperature range. The concentration of diffusing species is very small in comparison to other chemical species, the concentration of species far from the wall, C_∞ is infinitesimally small [1]. All thermo physical properties are assumed to be constant except the density in the buoyancy terms of the linear momentum equation which is approximated according to the Boussinesq's approximation. Under these assumptions, the equations that describe the physical situation are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u + g\beta(T - T_\infty) + g\beta^*(C - C_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\sigma B^2}{\rho c_p}u^2, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}, \tag{4}$$

where B^2 is the strength of magnetic field, β is the thermal expansion coefficient, β^* is the volumetric with solutal expansion coefficient and σ is the electrical conductivity. The second term on the right hand side of equation (3) represents Ohmic heating effect and the last term indicates viscous dissipation and D is the effective diffusion coefficient.

The boundary conditions are

$$u = 0,\, v = v_0,\, C = C_w,\, T = T_w \text{ at } y = 0,$$

$$u = u_c,\, C = C_\infty,\, T = T_\infty \text{ as } y \to \infty. \tag{5}$$

Following the lines of Chien-Hsin [3], the following changes of variables are introduced

$$\psi(x,\, y) = \sqrt{(4u_c \nu x)}, \tag{6}$$

$$\eta(x,\, y) = y\sqrt{\frac{u_c}{4\nu x}}. \tag{7}$$

The velocity components are given by

$$u = \frac{\partial \psi}{\partial y},\, v = -\frac{\partial \psi}{\partial x}. \tag{8}$$

Having introduced the stream function in equations (6) and (8), the continuity equation (1) is identically satisfied. Introducing the non dimensional variables and parameters:

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \tag{9}$$

$$\phi = \frac{\boldsymbol{C} - \boldsymbol{C}_\infty}{\boldsymbol{C}_w - \boldsymbol{C}_\infty}, \tag{10}$$

$$M = \frac{\sigma B^2 x}{\rho u_c} \text{ (Magnetic Parameter)}, \tag{11}$$

$$\mathrm{Pr} = \frac{\mu c_p}{K} \text{ (Prandtl number)}, \tag{12}$$

$$\mathrm{Sc} = \frac{\nu}{D} \text{ (Schmidt number)}, \tag{13}$$

$$N = \frac{\beta^*(C_w - C_\infty)}{\beta(T_w - T_\infty)} \text{ (Buoyancy ratio)}, \tag{14}$$

$$\mathrm{Ec} = \frac{u_c^2}{c_p(T_w - T_\infty)} \text{ (Eckert number)}, \tag{15}$$

$$S = -\boldsymbol{v}_0\sqrt{\frac{\boldsymbol{4x}}{\nu \boldsymbol{u}_c}} \text{ (Suction Parameter)}, \tag{16}$$

the equations (2) to (4) become

$$f''' + 3f\,f'' - 2f'^2 - 4M\,f' + 4(\theta + N\phi) = 0, \tag{17}$$

$$\frac{\theta''}{\mathrm{Pr}} + 3f\theta' - 4f'\,\theta + 4M\mathrm{Ec}f'^2 = 0, \tag{18}$$

$$\frac{\phi''}{\mathrm{Sc}} + 3f\phi' - 4f'\phi = 0. \tag{19}$$

With boundary conditions

$$\begin{aligned} &\eta = 0 : f(0) = \frac{S}{3},\ f'(0) = 0,\ \theta(0) = 1,\ \phi(0) = 1, \\ &\eta \to \infty : f'(\infty) = 1,\ \theta(\infty) = 0,\ \phi(\infty) = 0, \end{aligned} \tag{20}$$

equations (17) to (19) with boundary conditions (20) are integrated using Runge-Kutta and Shooting methods. Temperature and concentration are studied for different values of magnetic effects with buoyancy ratio between species and thermal diffusion at wall surface.

3. Results and Discussion

In order to get a clear insight of the physical problem, the velocity, temperature and concentration have been discussed by assigning numerical values to parameters encountered in the problem. To be realistic, the values of Schmidt number (Sc) are chosen for water vapour $(\mathrm{Sc} = 0.62)$ at temperature $25°\mathrm{C}$ and one atmospheric pressure. The values of Prandtl number is chosen to be $\mathrm{Pr} = 0.71$ which represents air at temperature $20°\mathrm{C}$ and one atmospheric pressure. The effect of buoyancy is significant for $\mathrm{Pr} = 0.71$ (air) due to the lower density of air that makes it more sensitive to the buoyancy forces. Eckert number is chosen to be $\mathrm{Ec} = 0.01$. The study of buoyancy ratio between species and thermal diffusion ($N = 1,\ 3$ and 5) effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reaction and the magnetic parameter is chosen to be $M = 1,\ 3$ and 5. The suction or injection parameter at the wall of the surface is chosen to be suction if $S > 0$ and injection if $S < 0$. Numerical results are displayed with the help of graphical illustrations. In the absence of viscous dissipation, the results have been compared with that of previous work [6] and it is found that they are in good agreement. The numerical results obtained are illustrated by means of Figures 2-7.

Effects of the buoyancy ratio with uniform magnetic field at the wall of the surface over the velocity, temperature and concentration are shown through Figures 2-4.

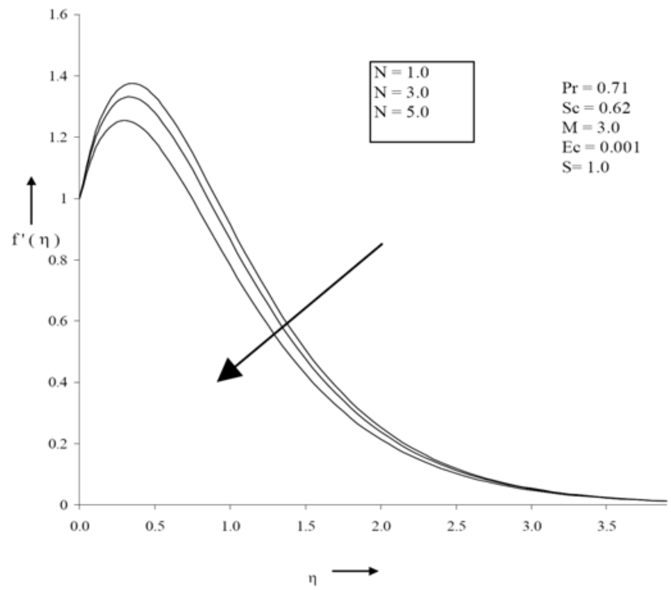

Figure 2. Effects of Buoyancy force on the velocity profiles.

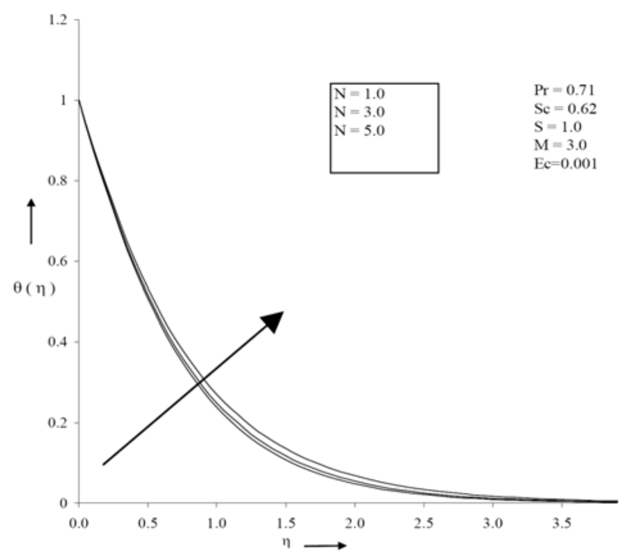

Figure 3. Buoyancy force over the temperature profiles.

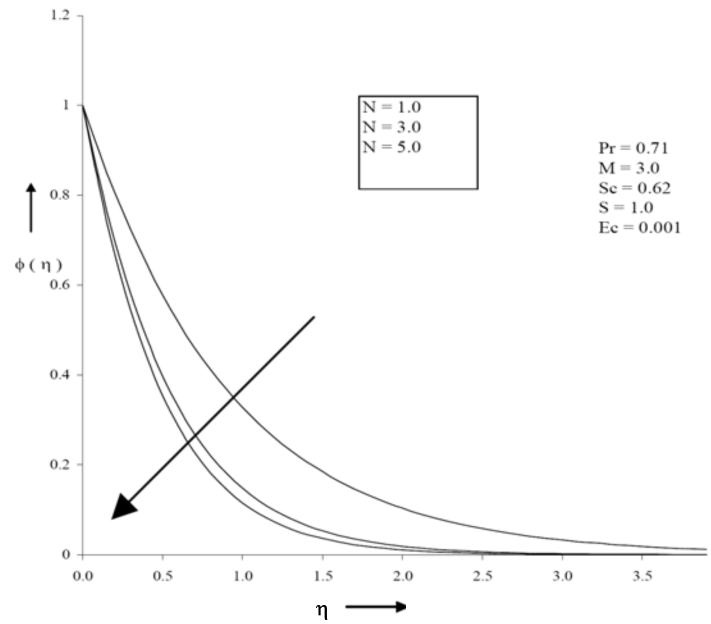

Figure 4. Buoyancy force on the concentration profiles.

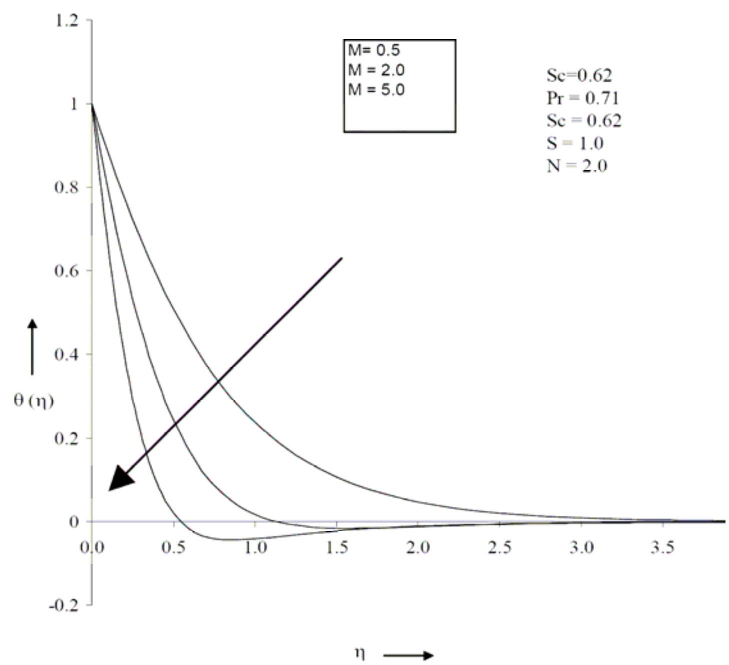

Figure 5. Magnetic effect over the temperature profiles.

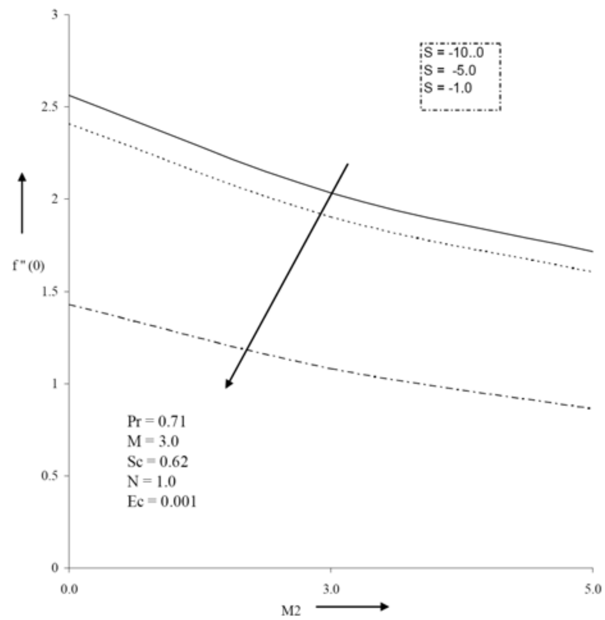

Figure 6. Effects of injection over the skin-friction.

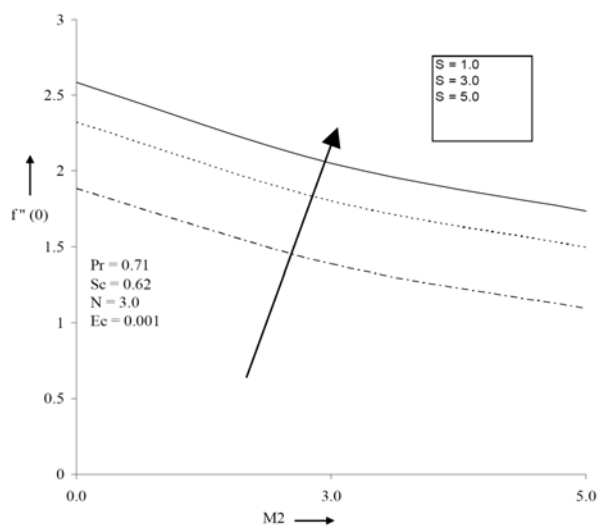

Figure 7. Effects of suction over the skin-friction.

In the presence of a fixed Pr and Sc number with constant magnetic field, it is clear that the velocity and the temperature of the fluid increase and the concentration of the fluid decreases with an increase of buoyancy ratio and these are displayed through Figures 2, 3 and 4, respectively. So, in the case of a fixed Prandtl and Schmidt number, the increase in buoyancy ratio accelerates the fluid motion and temperature of the fluid along the surface and decelerates the concentration of the fluid along the surface. All these physical behavior are due to the combined effects of Ohmic heating at wall surface with uniform magnetic field.

Figure 5 represents the dimensionless temperature profiles for different values of the magnetic parameter. In the case of uniform suction at the wall of surface, it is seen that the temperature of the fluid decreases with increase of magnetic parameter. In particular, the temperature of the fluid gradually changes from higher value to the lower value only when the strength of the magnetic field is higher than the species concentration effect. For large magnetic strength mechanism, interesting result is the large distortion of the temperature field caused for $M = 5.0$. Negative value of the temperature profile is seen in the outer boundary region for $M = 5.0$ and $N = 1.0$. All these physical behavior are due to the combined effect of buoyancy ratio between species and thermal diffusion and the strength of the magnetic field along with uniform suction at wall surface.

The effects of injection and suction in the presence of magnetic field on skin-friction are portrayed through Figures 6 and 7, respectively. It is noted that the skin-friction decreases with increase of injection and increases with increase of suction at wall surface.

4. Conclusion

This paper studied the combined heat and mass transfer of an electrically conducting fluid in MHD natural convection adjacent to a vertical surface is analyzed, taking into account the effects of Ohmic heating in the presence of suction or injection. The governing equations are approximated to a system of nonlinear ordinary differential equations by similarity transformation. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. Comparisons with previously published works are performed and excellent agreement between the results is obtained. The results are presented graphically and the conclusion is drawn that the flow field and other quantities of physical interest are significantly influenced by these parameters.

* In the presence of a fixed Pr and Sc number with constant magnetic field, the increase in buoyancy ratio accelerates the fluid motion and temperature of the fluid along the surface and decelerates the concentration of the fluid along the surface. All these physical behavior are due to the combined effects of Ohmic heating at wall surface with uniform magnetic field.

* Due to the uniform buoyancy ratio with constant suction at the wall of surface, it is clear that the temperature of the fluid decreases with increase of magnetic parameter. In particular, the temperature of the fluid gradually changes from higher value to the lower value only when the strength of the magnetic field is higher than the species concentration effect. For large magnetic strength mechanism, interesting result is the large distortion of the temperature field caused for $M = 5.0$. Negative value of the temperature profile is seen in the outer boundary region for $M = 5.0$ and $N = 1.0$. All these physical behavior are due to the combined effect of buoyancy ratio between species and thermal diffusion and the strength of the magnetic field along with uniform suction at wall surface.

* It is noted that the skin-friction decreases with increase of injection and increases with increase of suction at wall surface.

* Comparison of velocity profiles shows that the velocity increases near the plate and thereafter remains uniform. It is interesting to note that due to increase in buoyancy ratio, the concentration decreases at a faster rate in comparison to other parameters in the problem.

The analysis of the present investigation of flow through a wedge medium is playing a predominant role in the applications of Science and Engineering. The flow of this kind has enormous importance in technical problems such as flow through packed-beds, sedimentation, environmental problem, centrifugal separation of particles, blood rheology and in many transport processes in nature and Engineering devices, nuclear reaction, electronic equipments etc., in which the effect of buoyancy forces on the forced flow, or the effect of forced flow on the buoyant flow is significant. Particularly, the findings may be useful for the study of movement of oil or gas and water through the reservoir of an oil or gas field, in the migration of underground water and in the filtration and water purification processes.

Nomenclature

B^2	Magnetic field of strength.
C	Species concentration in the fluid.
C_∞	Species concentration with fluid away from the surface.
C_w	Species concentration near the surface.
C_p	Specific constant at constant pressure J/kgK.
D	Chemical molecular diffusivity m^2/s.
K	Thermal conductivity W/mK.
Pr	Prandtl number.
Sc	Schmidt number.
T	Temperature of the fluid.
T_w	Temperature of the wall.
T_∞	Temperature far away from the wall.
u, v	Velocity components m/s.
α	Thermal diffusivity m^2/s.
ν	Kinematic viscosity m^2/s.
ρ	Density of the fluid kg/m^3.

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