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RESEARCH ARTICLE

NORMAL, REGULARITY AND NEIGHBOURHOOD IN GENERALISED β^* CLOSED MAPS

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ABSTRACT

The In this paper we introduce a new class of β^*-continuous mapping and studied some of its basic properties. We obtain some characterizations of such functions. Moreover we define sub minimal structure and further study certain properties of β^*-closed sets, normal and regularity, further we study β^* open sets and β^* neighborhood.

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INTRODUCTION

Levine [9] introduced the concept of g-closed sets and studied their properties. A subset A of a space X is g-closed if and only if $cl(A) \subset O$ whenever A $\subset$ O and O is open. Hence every closed set is a g-closed set. The union and intersection of two g-closed set is g-closed set. Regular open sets and stronger regular open sets have been introduced and investigated by Stone [19] and Tang [21] respectively. Complements of regular open sets and strong regular open sets are called regular closed sets and strong regular closed sets. Andrijecvic[1], Arya and Nour[2], Bhattacharya and Lahiri[5],Levine[9],[10],Mashour et al[13] and Njastad[17] introduced and investigated semi-preopen sets, generalized semi open sets, semi generalized open sets, generalized open sets, semi-open sets, pre-open sets, generalized open set, semi-open sets pre-open sets and α-open sets which are some of the weak forms of open sets and the complements of theses sets are called the same types of closed sets respectively. Ganster and Reilly [8] have introduced locally closed sets which are weaker than both open and closed sets. Cameron [6] has introduced regular semi-open sets which are weaker than regular open sets. β^* open sets and β^* continuous functions were already introduced by Palanimani and Parimelazhagan, further the closed maps were studied.

Preliminaries

In this section we begin by recalling some definitions and properties Let (X, τ) be a topological spaces and A be a subset. The closure of A and interior of A are denoted by $cl(A)$ and $\text{int}(A)$ respectively. We recall some generalized open sets.

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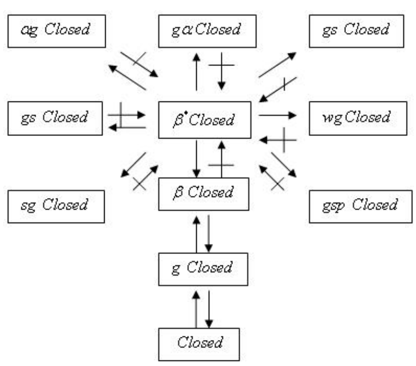
Definition [9] 2.1: A subset A of a space X is g-closed if and only if cl(A) $\subset$ G whenever A $\subset$ G and G is open.
Definition [20]2.2: A map f : X → Y is called g-closed if each closed set F of X, f(F) is g-closed in Y .
Definition[18]2.3: A map f : X → Y is called semi-closed if each closed set F of X, f(F) is semiclosed in Y.
Definition [15] 2.4 : A map f : X → Y is called α-open if each open set F of X, f(F) is α-set in Y.
Definition [7]2.5 : A map f : X → Y is called pre-closed if for each closed map F of X, f(F) is preclosed in Y.
Definition [12]2.6: A map f : X → Y is called regular-closed if for each set F of X, f(F) is regular closed in Y.
Definition (11)2.7: A map f : X → Y is said to be strongly continuous if $f^{-1}(V)$ is both open and closed in X for each subset V of Y .
Definition [4] 2.8: A map f: X → Y is said to be generalized continuous if $f^{-1}(V)$ is g-open in X for each set V of Y
Definition [15] 2.9: A subset A of a topological space X is said to be β^* closed set in X if cl(int(A)) contained in U whenever U is G-open
Remark 2.11: The following implications were well known



3.Properties of β^* closed sets In this section we study some of the properties of β^* closed set

Definition 3.1: A map f : X → Y is called β^* closed map if for each closed set F of X, f(F) is β^* closed set.

Remark 3.2: Every g-closed map is a β^* closed map and the converse is need not be true from the following example.

Example3.3: Let X = {a, b, c} and τ ={ϕ, x, {a}, {a, b}}, τ^C = { ϕ ,X, {b,c}, {c}} be topologies on X. f : X → Y each closed set f(F) is g-closed. Here {a, c} is g-closed but not β^* -closed.

Theorem 3.4: A map f : X → Y is β^* closed if and only if for each subset S of Y and for each open set U containing $f^{-1}(S)$ there is a β^* -open set V of Y such that S⊂ V and $f^{-1}(V) \subset U$

Proof: Suppose f is β^* closed. Let S be a subset of Y and U is an open set of X such that $f^{-1}(S) \subset$ U,Then V = Y − f^{-1}(X −U)is a β^* -open set V of Y Such that S ⊂ V such that $f^{-1}(V) \subset$ U.

For the converse suppose that F is a closed set of X. Then f^{-1}(Y −f(F)) ⊂ X −F and X −F is open. By hypothesis there is β^* -open set V of Y such that Y − f(F) ⊂ V and $f^{-1}(V) \subset$ X − F. Therefore F⊂X − f^{-1}(V).Hence Y − V⊂ f(F) ⊂ f(X − f^{-1}(V)) ⊂Y−V which implies f(F) = Y − V. Since Y − V is β^* -closed if f(F) is β^* -closed and thus f is a β^* -closed map.

Theorem 3.5: If f : X → Y is continuous and β^* -closed and A is a β^* -closed set of X then f(A) is β^* -closed.

Proof: Let f(A) ⊂ O where O is an open set of Y. Since f is g-continuous, $f^{-1}(O)$ is an open set containing A. Hence cl(int(A)) ⊂ $f^{-1}(O)$ is A is β^* -closed set. Since f is β^* -closed, f(cl(int(A))) is a β^* -closed set contained in the open set O which implies than cl(int(f(cl(int(A))))) ⊂ O and hence cl(int(f(cl(int(A))))) ⊂ O .f is a β^* -closed set.

corollary 3.6: If f : X → Y is g-continuous and closed and A is g-closed set of X the f(A)is β^* -closed.

Corollary 3.7: If f : X → Y is β^* -closed and continuous and A is β^* -closed set of X then

f_A : A → Y is continuous and β^* -closed set.

Proof: Let F be a closed set of A then F is β^* closed set of X. From above theorem 3.5 follows that $f_A(F)$ = f(F) is β^* -closed set of Y. Here f_A is β^* -closed and continuous.

Theorem 3.8: If a map f : X → Y is closed and a map g : Y → Z is β^* -closed then f : X → Z is β^* -closed.

Proof: Let H be a closed set in X. Then f(H) is closed and (g ∘ F)(H) = g(f(H)) is β^* -closed as g is β^* -closed. Thus g ∘ f is β^* -closed.

Theorem 3.9: If f : X → Y is continuous and β^* -closed and A is a β^* -closed set of X then f_A : A → Y is continuous and β^* -closed.

Proof: If F is a closed set of A then F is a β^* closed set of X. From Theorem 3.4, It follows that $f_A(F)$ = f(F) is a β^* -closed set of Y. Hence f_A is β^* -closed. Also f_A is continuous.

Theorem 3.10: If f: X → Y is β^* -closed and A = $f^{-1}(B)$ for some closed set B of Y then f_A : A → Y.is β^* -closed .

Proof: Let F be a closed set in A. Then there is a closed set H in X such that F = A ∩ H. Then $f_A(F)$ = f(A ∩ H) = f(H) ∩ f(B). Since f is β^* -closed. f(H) is β^* -closed in Y. so f(H) ∩ B is β^* -closed in Y. Since the intersection of a β^* -closed and a closed set is a β^* -closed set. Hence f_A is β^* -closed.

Remark 3.11: If B is not closed in Y then the above theorem does not hold from the following example.

Example 3.12: Take B = {b,c}. Then A = $f^{-1}(B)$ = {b, c} and {c} is closed in A but f_A({b}) = {b} is not β^* -closed in Y .{a} is also not β^* -closed in B.

4. Normal and Regularity

In this section we introduce the new class of β^* -regular and studied some of its properties.

Theorem 4.1: If f: X → Y is continuous , β^* closed map from a normal space X onto a space Y then Y is normal.

Proof: Let A, B be disjoint closed sets in Y. Then $f^{-1}(A)$, f−1(B) are disjoint closed sets of X. Since X is normal, there are disjoint open sets U, V in X such that $f^{-1}(A) \subset$ U and $f^{-1}(B) \subseteq$V. Since f is β^* -closed by theorem 3.4, there are β^* -open sets G,H in Y such that A ⊂ G,B⊂H and $f^{-1}(G)$ ⊂U and $f^{-1}(H)$ ⊂V. Since U, V are disjoint. int G, int H are disjoint open sets.Since G is β^* -open, A is closed and A ⊂G,A⊂ cl(int(G)). similarly B ⊂ cl(int(H)). Hence Y is normal.

Theorem 4.2: If f : X → Y is an open continuous β^* -closed surjection, where X is regular then Y is regular.

Proof: Let U be an open set containing a point P in Y. Let X be a point of X such that f(X) = P. Since X is regular and f is continuous there is an open set U such that $x \in$ V ⊆ cl(int(V)) ⊆ f^{-1}(V). Hence P ∈ f(V) ⊂ f(cl(int(V)) ⊂U. Since f is β^* closed f(cl(int(V))) is β^* -

closed set contained in the open set U. It follows that cl(int(f(cl(int(V))))$\subset$U and hence p$\in$ f(V)$\subset$ cl(int(f(V)))$\subset$U and f(V) is open. Since f is open. Hence Y is regular.

Remark 4.3: The normality is preserved under regular closed, continuous and surjective.

Example 4.4: In the example 3.12. It is shown that f is β^*-closed {b,c} is a regular closed set in (X,τ_1)and it is not closed in (X,τ_2).Hence f is not regular closed.

Example 4.5: Let τ_1 be the countable complement topology on the real line R and τ_2 be the usual topology on R and f : (R, τ_1) → (R, τ_2) be the identity map. Then f is regular closed by the remark immediately after the above example. But f is not β^*-closed. If

A = {1/n, n$\in$N} then A is closed in (R, τ_1) and f(A) = A is not β^*-closed as f(A)$\subset$(0, 2) and (0, 2) is open in (R, τ_2).But cl(int(f(A))) $\subset$ (0, 2).

Theorem 4.6: If A is β^*-closed set of a space X then IndA ≤ IndX

Proof: It suffices to show that if IndX ≤ n and A is β^*-closed set of X then IndA ≤ n. We prove this theorem by induction. The result holds trivially for n=1. Assume that for every β^*-closed set A of X , ind X ≤ n − 1 ⇒ Ind A≤ n − 1.

Let X be space with Ind ≤ n. Let A be a β^* closed set of X. Let E be a closed set of A and G be an open set of A such that E$\subset$G. Then there exist a closed set F of X and an open set H of X such that E = A ∩ F and G = A ∩ H. Since E is closed in A and A is β^*-closed. Since Ind X ≤ n, there is an open set V of X such that cl(int(E))$\subset$V$\subset$H and Indbd(V)≤ n − 1. Then V ∩ A is an open set of A such that E$\subset$V ∩ A$\subset$G and bd_A(V ∩ A) $\subset$bd(V). Now

bd_A(V ∩ A) is a β^*-closed set of bd(V).By induction hypothesis and Indbd_A(V ∩A) ≤ n−1. Hence IndA ≤ n.

Theorem 4.7: If A is a β^*-closed set of a space X then dim A ≤ dim X.

Proof: If dimX = 0 then dimA ≤ 0 = dim X. Hence dimA ≤ dimX. If dimX ≤ 0 then dimX = n, where n is an integer greater than or equal to -1. If n = −1 dimX = −1which implies that

X =ϕ and hence A = ϕ and dimA = −1 = dimX and thus dimA ≤ dimX.

Next suppose dimX = n where n ≥ −1 and let A be a β^*-closed set of X. Let {u_1, u_2, u_3, ...u_k} be a finite open cover of A. Then for i = 1, 2, 3, ...K there exist open sets.V_1 of X such that u_1 = A ∩ V_1. Since A is β^*-closed and $U_k^{i=1} v_i$ is an open set containing A, cl(int(A))$\subset$ $U_k^{i=1} pv_i$ Since cl(int(A)) is a closed set, dimcl(int(A)) ≤ n ,so the finite open cover {cl(int(A∩ v_i, i = 1, 2, 3, ..k} cl(int(A)) has a refinement cl(int(A)) ∩ w_i, i = 1, 2, 3, ..k or order at most n+1, where each w_1 is open in X and cl(int(A)) ∩ w_1 $\subset$ cl(int(A)) ∩ V_i for each i. Then {A ∩ w_i) : i = 1, 2, ...} is an open cover of A refining {u_i, i = 1, 2, 3, ...k} and of order not exceeding n + 1. Hence dimA ≤ n which implies that dimA ≤dimX.

Theorem 4.8: If A is a β^*-closed set of a space X then DindA ≤ DindX.

Proof : Let X be a space such that DindX = n and A be a β^*-closed set of X. By using the notations of the above thoerem, cl(int(A))$\subset$ $\cup$$V_i$. Since cl(int(A)) is a closed set, DindA ≤ n. Hence for every open cover V_i ∩ cl(intA)), i = 1, 2, 3...k there is a disjoint family W_i,J =1,2,3, ...k of open sets cl(int(A)) refining V_i ∩ cl(int(A)) i = 1, 2, 3, ...k and such that Dind(cl(int(A))- $U_{j=1}^k w_j$ ≤ n−1. But A $-U_{j=1}^k w_j \subset$ cl(int(A))− $U_{j=1}^k w_j$ and A$-U_{j=1}^k w_j$ = A∩(cl(int(A))− $U_{j=1}^k w_j$ is

a β^*-closed set of cl(int(A)) as the intersection of β^*-closed set . By induction hypothesis

Dind(A$-U_{j=1}^k w_j$) ≤ n−1. Also W_j ∩ A, j = 1, 2, 3...k is a disjoint family of open sets of A refining u_1, u_2, u_3, ...u_k Thus DindA ≤ n and the theorem is proved

5. β^* Open sets and β^* Neighborhoods

In this section we introduce β^* neighborhoods (β^*-nbhd) topological spaces by using the notion of β^* open sets and study some properties .

Definition 5.1: Let X be the point in topological space X , then the set of all β^*-

neighborhood of a X is called β^*-nbhd system of X which is denoted by β^*-N(X)

Theorem 5.2 : Let X be the topological space and each $x \in X$ .Let $\beta^* - N(x,\tau)$ be the collection of all β^*-nbhd of X .then we have the following results

(i) $\forall x \in X, \beta^* - N(X) \neq \phi$

(ii) $N \in \beta^* - N(X) \Rightarrow x \in N$

(iii)

$$N \in \beta^* - N(X), M \supset N \Rightarrow M \in \beta^* - N(X)$$

(iv) $N \in \beta^* - N(X) \Rightarrow \exists\ M \in \beta^* - N(X)$ such that M $\subset$ N and $M \in \beta^* - N(Y), \forall Y \in M$

Proof : (i) Since X is β^* open set ,it is β^*-nbhd of every $\forall x \in X$, Hence there exists atleast one β^*-nbhd (namely X) for each $x \in X$ .Hence $\forall x \in X, \beta^* - N(X) \neq \phi$

(ii) if $N \in \beta^* - N(X)$,then N is a β^*-nbhd of x .then by definition β^*-nbhd(x)$\in$N

(iii) Let $N \in \beta^*$-nbhd and M $\supset$ N, then there is a β^*-open set U such that $x \in U \subset N$

Since N$\subset$M , $x \in U \subset M$ and M is β^*-nbhd of X ,Hence $M \in \beta^* - N(X)$

(iv) If N $\in \beta^* - N(X)$,then there exists a β^*-open set such that $x \in M \subset N$,since M is a β^*-open set ,it is β^*-nbhd of each of its points. Therefore $M \in \beta^* - N(Y)$ for every $Y \in M$

Theorem 5.3: Let X be a nonempty set, for each x $\in$ X, let β^*-N(x) be nonempty collection of subsets of X satisfying following conditions.

(i) N $\in \beta^*$-N(X, τ)) $\Rightarrow x \in N$.

(ii) Let τ consists of the empty set and all those non-empty subsets of U of X having the property that x $\subset$ U implies that there exists an N $\subset \beta^*$-N(X) such that x $\in$ N$\subset$ U, Then τ is a topology for X.

Proof : (i) φ $\in \tau$ by definition. We now show that x $\in \tau$. Let x be any arbitrary element of X. Since β^*-N(x) is non empty, there is an N$\in \beta^*$-N(X) and so x$\in$ N.

Since N is a subset of X, we have x $\in$ N $\in$ X. Hence X $\in \tau$.

(ii) Let $U_\lambda \in \tau$ for every λ $\in$ Λ. If x $\in$ U {U_λ : λ $\in$ Λ}, then x $\in$ $U_{\lambda x}$ for some λx$\in$Λ.

Since $U_{\lambda x} \in \tau$, there exists an N $\in \beta^*$-N(x) such that x $\in$N$\in$ Uλx and consequently

x $\in$ N$\in$ U {Uλ : λ $\in$Λ}. Hence U { Uλ : λ $\in$ Λ} $\in \tau$ .It follows that τ is topology for X.

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