Haar' s Measure using Triangular Fuzzy Finite Topological Group

G. Veeramalai, P.Ramesh

Abstract: ***In this paper, A new approach is used to apply Haar's measure theory to triangular fuzzy number theory for comprehending and generalizing the uniqueness of invariant measure when there are uncertainty and risk. If*** $\tilde{T}$ ***is a triangular fuzzy finite Topological group and*** $\tilde{X}$ ***is its subgroup,*** $\tilde{X}$ ***also being a triangular fuzzy number, then*** $\frac{\mu(\tilde{x})}{\mu(\tilde{T})}$

Keywords: Haar's measure, Invariant measure, Topological group, Triangular fuzzy number etc.

I. INTRODUCTION

This Invariant measure plays an essential role in numerous fields of mathematical sciences, for instance, the uncertainty principle identified with Probability introduced in (R.M Dudley, 2002) does exclude any announcement around an invariant measure, but rather the measure assumes a critical part in demonstrating the probability theorem, as appearing in this paper, the structure of probability additionally offer ascent to probability distributions invariant measure. It is fascinating to perceive the method of generalizing the measure theory to probability distributions.

The general hypothesis of measure and integration was conceived in the mid twentieth century. It is currently a vital tool in significantly various fields of mathematical sciences, including functional analysis, partial differential equations, harmonic analysis, probability theory and dynamical frameworks. Surely, it has turned into a combined theory. Many different topics can agreeably accompany a treatment of this theory. The companionship between integration and functional analysis and, in particular, between integration and weak convergence, has been fostered here: this is important, for instance, in the analysis of nonlinear partial differential equations.

II. PRELIMINARIES

1. Definition:

An invariant is the one which has no change under a set of transformations.

Revised Manuscript Received on September 2, 2019.

[1]**G. Veeramalai,** Department of Mathematics,
M.Kumarasamy College of Engineering, (Autonomous), Karur, Tamil Nadu, India, Email: veeramusiri@gmail.com
[2]**Dr. P.Ramesh,** Dean and HOD, Department of Science and Humanities, M.Kumarasamy College of Engineering, (Autonomous), Karur, Tamil Nadu, India, Email: veeramusiri@gmail.com

2. Triangular Fuzzy Number

The fuzzy system of numeration that typically utilized in applications is that the triangular (shaped) fuzzy numbers [8].

2.1 Fuzzy set: [3]

A fuzzy set L must the three axioms,

i . $\tilde{L}$ is a ordinary set.

ii. ${}^{\alpha}\tilde{L}$ is closed interval , for all $\alpha \in [0,1]$

iii. $\tilde{L}$, ${}^{0+}\tilde{L}$ is bounded.

2.2 Triangular Fuzzy Number: [13]

A fuzzy numbers delineated with three points as:

$\tilde{L} = (l_1, l_2, l_3)$

This illustration is taken as membership rule and holds the subsequent axioms

(i) Increasing function is l_1 to l_2

(ii) Decreasing function is l_2 to l_3

(iii) $l_1 \le l_2 \le l_3$

$$\mu_L(x) = \begin{cases} 0, & for\ x < l_1 \\ \dfrac{x - l_1}{l_2 - l_1} & for\ l_1 < x < l_2 \\ \dfrac{l_3 - x}{l_3 - l_1} & for\ l_2 < x < l_3 \\ 0, & for\ x > l_3 \end{cases}$$

2. 3. A Triangular fuzzy number is positive is defined as $\tilde{L} = (l_1, l_2, l_3)$, here $l_i > 0, i = 1,2,3$

2.4 A Triangular fuzzy number is negative is defined as $\tilde{L} = (l_1, l_2, l_3)$, here $l_i < 0, i = 1,2,3$

2.5. Two triangular fuzzy numbers $\tilde{L}$ and $\tilde{M}$ are identically equal, that is $\tilde{L} = \tilde{M}$, if and only if $l_1 = m_1$, $l_2 = m_2$ and $l_3 = m_3$

III. HAAR'S MEASURE

3.1 Definition of Left Haar's measure:

A Left Triangular fuzzy Haar's measure $\tilde{\mu}$ on topological group $\tilde{T}$ is Radon measurable and is invariant under left translation

(i.e) $\mu(\tilde{t}\tilde{x}) = \mu(\tilde{x}) \qquad \forall \tilde{t} \in \tilde{T}$

$$\mu(\tilde{t}\tilde{x}) = \mu((t_1t_2t_3)(x_1x_2x_3)) = \mu((t_1x_1, t_2x_2, t_3x_3))$$

$$= \mu(t_1x_1)\mu(t_2x_2)\mu(t_3x_3)$$
$$= \mu(x_1)\mu(x_2)\mu(x_3)$$
$$= \mu(x_1x_2x_3)$$
$$= \mu(\tilde{x})$$

3.2 Definition of Right Haar's measure:

A Right Triangular fuzzy Haar's measure μ on topological group $\tilde{T}$ is Radon measurable and is invariant under right translation

(i.e) $\mu(\tilde{x}\tilde{t}) = \mu(\tilde{x}) \qquad \forall \tilde{t} \in \tilde{T}$

where $\tilde{x}$ *and* $\tilde{T}$ are triangular fuzzy numbers.

$$\mu(\tilde{x}\tilde{t}) = \mu((x_1x_2x_3)(t_1t_2t_3)) = \mu((x_1t_1, x_2t_2, x_3t_3))$$

$$= \mu(x_1t_1)\mu(x_2t_2)\mu(x_3t_3)$$
$$= \mu(x_1)\mu(x_2)\mu(x_3)$$
$$= \mu(x_1x_2x_3)$$
$$= \mu(\tilde{x})$$

3.3 Lemma:

The measure of a subgroup of a Triangular fuzzy invariant finite topological group is invariant

Assumption: Let Triangular fuzzy Topological group be finite

Let Triangular fuzzy invariant finite topological group be $\tilde{T}$

$\mu(\tilde{a}\tilde{T}) = \mu(\tilde{T}\tilde{a}) = \mu(\tilde{T})$ by Triangular fuzzy Haar's measure

Identity:

The identity element of $\tilde{T}$ be $'\tilde{e}'$

Here $\tilde{e}$ is the triangular number so, $\tilde{e} = (e_1e_2e_3)$

$$\mu(\tilde{e}\tilde{x}) = \mu(\tilde{x}\tilde{e}) = \mu(\tilde{x})$$

$$\mu(\tilde{e}\tilde{x}) = \mu((e_1, e_2, e_3)(x_1, x_2, x_3)) = \mu((e_1x_1, e_2x_2, e_3x_3))$$

$$= \mu(e_1x_1)\mu(e_2x_2)\mu(e_3x_3)$$
$$= \mu(x_1)\mu(x_2)\mu(x_3)$$
$$= \mu(x_1x_2x_3)$$
$$= \mu(\tilde{x})$$

$$\mu(\tilde{x}\tilde{e}) = \mu((x_1, x_2, x_3)(e_1, e_2, e_3)) = \mu((x_1e_1, x_2e_2, x_3e_3))$$

$$= \mu(x_1e_1)\mu(x_2e_2)\mu(x_3e_3)$$
$$= \mu(x_1)\mu(x_2)\mu(x_3)$$
$$= \mu(x_1x_2x_3)$$
$$= \mu(\tilde{x})$$

$$\mu(\tilde{e}\tilde{x}) = \mu(\tilde{x}\tilde{e}) = \mu(\tilde{x})$$

So identity satisfied.

Therefore the triangular fuzzy measurable group of a subgroup is invariant.

3.4 Theorem:

Measure of a subgroup of a triangular fuzzy finite topological group divides the measure of the groups.

Proof:

Consider $\tilde{x}$ as the subgroup of $\tilde{T}$ (Here $\tilde{x} \le \tilde{T}$)

and let $\tilde{x}$ *of* $\tilde{T}$ be finite

If i) $\tilde{x} = \tilde{T}$ it is obviously proved.

ii) $\tilde{x} \ne \tilde{T}$

$$\mu(\tilde{x}) = \tilde{m} \ , \ \mu(\tilde{T}) = \tilde{n}$$

$$\mu(x_1, x_2, x_3) = (m_1, m_2, m_3)$$

$$\mu(t_1, t_2, t_3) = (n_1, n_2, n_3)$$

Every Triangular fuzzy left Haar's measure is equal to right Haar's measure of x in T.

Since $\mu(\tilde{e}\tilde{x}) = \mu(\tilde{x})$

$$\mu(\tilde{e}\tilde{x}) = \mu((e_1e_2e_3)(x_1x_2x_3)) = \mu((e_1x_1, e_2x_2, e_3x_3))$$

$$= \mu(e_1x_1)\mu(e_2x_2)\mu(e_3x_3)$$
$$= \mu(x_1)\mu(x_2)\mu(x_3)$$
$$= \mu(x_1x_2x_3)$$
$$= \mu(\tilde{x})$$

as $\tilde{x}$ is the right Haar's measure of x in T.

Likewise $\mu(\tilde{x}\tilde{a}), \mu(\tilde{x}\tilde{b}), \mu(\tilde{x}\tilde{c}), \mu(\tilde{x}\tilde{d}),............$ are right Haar's measures of x in T

So

$$\mu(\tilde{x}\tilde{a}) = \mu(\tilde{x}\tilde{b}) = \mu(\tilde{x}\tilde{c}) = \mu(\tilde{x}\tilde{d}) = ... = \mu(\tilde{x}) = \tilde{m}$$

$$\mu((x_1x_2x_3)(a_1a_2a_3)) = \mu((x_1x_2x_3)(b_1b_2b_3)) = \mu((x_1x_2x_3)(c_1c_2c_3)) = ... = (m_1m_2m_3)$$

$$\Rightarrow \mu(x_1a_1, x_2a_2, x_3a_3) = \mu(x_1b_1, x_2b_2, x_3b_3) = \mu(x_1c_1, x_2c_2, x_3c_3) = ... = (m_1m_2m_3)$$

$$\mu(x_1a_1) = \mu(x_1b_1) = \mu(x_1c_1) = ... = m_1$$

$$\mu(x_3a_3)=\mu(x_3b_3)=\mu(x_3c_3)=.........=m_3$$

distinct Haar's measure of $\tilde{x}$ in $\tilde{T}$

Assume that $\tilde{k}$ be the number of division of $\tilde{T}$

Always the right Haar's measure of disjoint sets induces a

$$\mu(\tilde{T})=\mu(\tilde{x}\tilde{a})+\mu(\tilde{x}\tilde{b})+\mu(\tilde{x}\tilde{c})+........ \qquad [k\ \ times]$$

$$\mu(t_1t_2t_3)=\mu((x_1x_2x_3)(a_1a_2a_3))+\mu((x_1x_2x_3)(b_1b_2b_3))+\mu((x_1x_2x_3)(c_1c_2c_3))+............$$

$$=\mu(x_1a_1,x_2a_2,x_3a_3)+\mu(x_1b_1,x_2b_2,x_3b_3)+\mu(x_1c_1,x_2c_2,x_3c_3)+.................$$

$$=\mu(x_1a_1)\mu(x_2a_2)\mu(x_3a_3)+\mu(x_1b_1)\mu(x_2b_2)\mu(x_3b_3)+\mu(x_1c_1)\mu(x_2c_2)\mu(x_3c_3)+.........$$

$$\mu(t_1)\mu(t_2)\mu(t_3)=[\mu(x_1a_1)+\mu(x_1b_1)+\mu(x_1c_1)+....],[\mu(x_2a_2)+\mu(x_2b_2)+\mu(x_2c_2)+......],$$
$$[\mu(x_3a_3)+\mu(x_3b_3)+\mu(x_3c_3)+......]$$

$$\mu(t_1)=[\mu(x_1a_1)+\mu(x_1b_1)+\mu(x_1c_1)+......... \quad k\,times]$$
$$\mu(t_2)=[\mu(x_2a_2)+\mu(x_2b_2)+\mu(x_2c_2)+......... \quad k\,times]$$
$$\mu(t_3)=[\mu(x_3a_3)+\mu(x_3b_3)+\mu(x_3c_3)+......... \quad k\,times]$$

$$n_1=m_1+m_1+m_1+.........ktimes \qquad \Rightarrow n_1=k_1m_1(k\ \ times of\, m_1)$$
$$n_2=m_2+m_2+m_2+.........ktimes \qquad \Rightarrow n_2=k_2m_2(k\ \ times of\, m_2)$$
$$n_3=m_3+m_3+m_3+.........ktimes \qquad \Rightarrow n_3=k_3m_3(k\ \ times of\, m_3)$$

$\mu(x_1)$ Divides $\mu(t_1)$

$\mu(x_2)$ Divides $\mu(t_2)$

$\mu(x_3)$ Divides $\mu(t_3)$

(i.e) $\mu(\tilde{x})Divides\,\mu(\tilde{T})$

Hence triangular fuzzy finite topological group divides the measure of the groups

IV. CONCLUSION

Applying Haar's measure theory to triangular fuzzy measure theory is simple to know and generalize the invariant uniqueness in the real life situations. Hence, Triangular fuzzy measure of a subgroup of a triangular fuzzy finite subgroups divides the measure of the groups

REFERENCES

1. Ahmady N., A Fuzzy Newton-Cotes method For Integration of Fuzzy Functions,ISPACS, Volume 2015, Issue 2, Year 2015 Article ID jfsva-00247, 8 Pages.
2. Buckley J.J., Y. Qu (1991). Solving fuzzy equations: a new solution concept. *Fuzzy Sets and System*. 39: 291-301.
3. Chang C.L. Fuzzy Topological Space. Journal of Mathematical Analysis and Applications 24 (1968), 182-190.
4. George J. Klir, Ute St. Clair, Bo Yuan (1997). *Fuzzy Set Theory: Foundations and Applications.* Prentice Hall PTR.
5. Goetschel R., W. Voxman (1986). *Elementary Calculus*, Fuzzy Sets and Systems 18:31-43
6. Levine N., Generalized closed sets in topology, Rend. Circ. Math. Palermo 19 (1970), 89-96.
7. Lowen R., " Fuzzy Topological Space and Fuzzy Compactness, Journal of Math. Anal. Appl. 56(1976), 621.633
8. Negoita, C.V. Ralescu D.A.: Applications of Fuzzy Sets to System Analysis, John wily and sons, New York 1975.
9. Nasseri S.H., Zahmatkesh F.: Huang method for solving fully fuzzy linear system of equations. The Journal of Mathematics and Computer Science, **1**, 1-5 (2010)
10. Rajasethupathy K.S.; S.Lakshmivarahan "Connectedness In fuzzy Topology" Volume.13(1977).
11. Simmons G.F.: Introduction to Topology and Modern Analysis. McGraw Hill, 1963.
12. Sun X., Guo S.: Solution to general fuzzy linear system and its necessary and sufficient condition. Fuzzy Information and Engineering **3**, 317-327 (2009)
13. Veeramalai G., "Unconstrained Optimization Techniques Using Fuzzy Non Linear Equations" AARJMD,ISSN: 2319- 2801.Volume 1, Issue 9, (May2013), PP 58-67
14. Veeramalai G. and P.Gajendran, "A New Approaches to Solving Fuzzy Linear System with Interval Valued Triangular Fuzzy Number" Indian Scholar An International Multidisciplinary Research e-Journal , ISSN: 2350-109X,.Volume 2, Issue 3, (Mar2016), PP 31-38
15. Veeramalai G.., A. Arunangel Priya : On Solving Three Variables Unconstrained Optimization Problem Using Triangular Fuzzy Systems, International Journal of Pure and Applied Mathematics, Volume 118 No. 20 2018, 2037-2044 (2018)
16. Wong C. K., covering properties of fuzzy topology spaces, j. Math. Anal. Appt. 43 (1973), 697-703.
17. Zadeh, L.A., The concept of a Linguistic variable and its applications to approximate reasoning – parts I, II and III", Inform. Sci. 8(1975) 199-249; 8 1975 301-357; 9(1976) 43-80.
18. Zadeh. L. A: Fuzzy sets . Information and control 8 (1965),3, 338-353

AUTHORS PROFILE

Veeramalai, G., is doing PhD in Mathematics from Periyar University, Salem. He has previously done M.Phil in Mathematics from Bishop Heber college-Trichy. He is currently working as an Assistant Professor in Department of Mathematics, M.Kumarasamy college of Engineering, Karur. . He has 9 years of experience in teaching. His research interests include Fuzzy topology, Interval analysis and differential equations. He has published more than 12 Papers in reputed journals. His objective is to achieve success in all paths, through continual learning with determination to pursue and achieve the heights of Teaching Profession. He can be contacted on email- veeramusiri@gmail.com

Dr. P.Ramessh, is currently working as a Dean and HOD, Department of Science and Humanities from 2005 onwards in M.Kumarasamy College of Engineering, Karur. He has 15 years of experience in teaching. He has written more than five books in Engineering Mathematics. His research interests include Fuzzy topology and differential equations. He received Ph.D degree in 2016 from Madurai Kamaraj University, Madurai. His object is Self-directed, enthusiastic educator with a passionate commitment to student development and the learning experience. Skilled in the design of challenging, enriching, and innovative activities that address the diverse interests and needs of students. He can be contacted at - vprkarur@gmail.com.