Sadayan Navaneethakrishnan* and Ayyanar Athijayamani

Analysis of the tensile properties of natural fiber and particulate reinforced polymer composites using a statistical approach

Abstract: In the present study, the effect of fiber and (5% wt) coconut shell powder (CSP) loading on the tensile properties of randomly oriented roselle fiber reinforced vinyl ester (RV) composites was carried out. Composite specimens were fabricated using the hand lay-up technique. It was observed that tensile properties of vinyl ester composites increase upon reinforcement with roselle fibers and CSP particles. The reinforcement of roselle fibers has significant effects on the tensile strength and modulus of the composite. The fractographic study was carried out on the surface of fractured composite specimens using scanning electron microscopy. A better interfacial adhesion, fiber dispersion and less fiber pull out on the surface of fractured composite specimens were identified. The tensile strength of RV composites has also been statistically analyzed by two-parameter Weibull distribution. Twenty-five tension tests were carried out. The results obtained varied between 37.89 MPa and 45.07 MPa. Furthermore, the inspection of the developed distribution was examined by the Kolmogorov-Smirnov (KS) test. The results show that the gained two-parameter Weibull distribution can be used to express the tensile strength and predict its values accurately.

Keywords: CSP; mechanical properties; polymer-matrix composites (PMCs); statistical properties/methods; two-parameter Weibull distribution.

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1 Introduction

Recently, plant-based lignocellulosic natural fibers have been an interesting, environmentally friendly alternative to the use of man-made synthetic fibers as reinforcement in engineering composites, because of the benefits that these fibers provide over conventional reinforcement materials [1, 2]. The potential of natural fiber-based composites using banana, bamboo, kenaf, jute, hemp, sisal, pineapple, coir, flax, vakka etc., as reinforcing fiber in a polymer resin matrix has received considerable attention among material scientists and researchers all over the world for their excellent specific properties [3–6]. The availability, renewability, low density, and price as well as satisfactory mechanical properties of bio natural fibers make them an attractive ecological alternative to glass, carbon and man-made fibers used for the manufacturing of composites [7–13]. Among the various natural fibers, roselle fibers (*Hibiscus sabdariffa* L.) possess moderately high specific strength and stiffness and hence, can be used as a reinforcing material in polymeric resin matrices to make useful structural composite materials. Roselle is native from India to Malaysia, where it is commonly cultivated, and must have been carried at an early date to Africa. It is a medicinal plant in the *Malvaceae* family. In South India regions, this plant is used for the safeguard of paddy and ground nut plant, etc. The fibers extracted from the stems of roselle plants are used to produce the rope which is used to deliver the water from deep wells. Several studies were carried out on the roselle fibers as reinforcement for polymer matrix [14–18].

Generally, polymer-matrix composite (PMC) materials have anisotropic characteristics and, therefore, have different mechanical properties in different directions. In addition to this, they present varying strengths due to their internal structure, which means that there is no specific strength value to represent their mechanical behavior. In particular, their strength properties are usually scattered due to their inhomogeneity and to the brittleness of the matrices and fibers. This leads to the necessity of employing statistical analysis for their safe utilization in design and manufacturing. Statistical analysis has recently been used for the determination of static and dynamic mechanical properties of ceramic matrix composites, metal matrix composites and PMCs [19–25].

The aim of this study is to investigate the effect of randomly oriented roselle fibers loading on the tensile properties of reinforced vinyl ester (RV) composites. The best set of fabrication parameters with maximum response value is selected from the experimental results for the use in statistical analysis. The variation of the tensile strength of composite specimens has also been modeled using two-parameter Weibull distribution. Twenty-five tests were performed, and using the test data, the corresponding two-parameter Weibull distribution was determined. A graphical method was used to estimate parameters. Furthermore, the inspection of the developed distribution was examined by the Kolmogorov-Smirnov (KS) test.

2 Materials and methods

2.1 Materials

Vinyl ester resin (bisphenol-A epoxy vinyl ester) was used as matrix material. It was obtained from GVR Chemicals Pvt. Ltd., Madurai, Tamilnadu, India under the trade name of Satyan Polymer. Methyl ethyl ketone peroxide ($C_8H_{18}O_6$), cobalt 6% naphthenate ($C_{20}H_{34}CoO_4$) and N-N dimethyl aniline ($C_8H_{11}N$) were used as accelerator, catalyst and promoter, respectively. Vinyl ester resins are somewhere between polyesters and epoxies in terms of mechanical properties. One of their major advantages is that they offer better resistance to moisture absorption and hydrolytic attack than polyester. They have excellent physical and mechanical properties and is familiar for its versatility as a composite matrix. The cast resin has a tensile strength of 29.1 MPa and tensile modulus of 1097.4 MPa.

Roselle fibers (*Hibiscus sabdariffa* L.) and coconut shell powders (CSP) are procured from the nearest village of the author's home town and used as reinforcement as received, without any treatment. The ranges of CSP (5% wt), roselle fiber loading taken for this study were 12.29 wt%, 23.97 wt%, 35.08 wt%, 45.67 wt% and 55.77 wt%. The fiber length was fixed as 6 mm. The physical properties of roselle fibers and vinyl ester resin are shown in Table 1.

2.2 Processing

Composites with different amounts of roselle fibers were obtained by using the hand lay-up technique developed in our laboratory. Prior to fabrication of composite specimens, the composite materials were prepared by mixing the resin and the roselle fibers in a mechanical stirrer. Before that, the mold was well cleaned and dried and a release agent was laid up on the two sides of the mold. The mixing of resin and fiber was poured into the mold of size of 150 mm×150 mm×3 mm, after adding the accelerator, catalyst and promoter. The curing was carried out for 24 h at room temperature. No macrovoids and macrocracks were observed in all of the samples. The composites were fabricated in the form of flat sheets of thickness 3 mm (Figure 1).

2.3 Characterization of composite specimens

The density of RV composites was determined experimentally by the simple water immersion technique. The densities of neat vinyl ester and composites were measured by measuring mass and volume. The void fraction of RV composites was also determined experimentally by soaking composite samples in kerosene oil. The density and void fraction of the composite samples will be obtained from following equations [Eq. (1) and Eq. (2)] by using Archimedes principle:

$$\text{Density}=\frac{W_d}{W_{so}-W_{su}} \tag{1}$$

$$\text{Void fraction}=\frac{W_{so}-W_d}{W_{so}-W_{su}} \tag{2}$$

where W_d (dry weight) is the weight of the sample at completely dried condition, W_{so} (soaked weight) is the weight

Table 1: Physical properties of roselle fiber and vinyl ester resin.

S. no	Vinyl ester resin		Roselle fiber	
1	Appearance	Clear yellow	Appearance	Gold brown
2	Viscosity, cps	400	Tensile strength (MPa)	165
3	Specific gravity	1.09	Young's modulus (GPa)	27.68
4	Styrene content (%)	32	% of elongation	0.65

cps, centipoise.

Figure 1: Digital image of composite specimens by hand lay-up method.

of the sample soaked in kerosene oil and W_{su} (suspended weight) is the weight of the sample suspended in the oil through a string.

The tensile tests were performed on the prepared composite samples as per the ASTM D 638–10 (2010) [26] using the Instron 3382 model universal testing machine (Shimadzu make) at 23±2°C temperature and relative humidity of 50±5 RH. The length, width and the thickness of each specimen were approximately 150 mm×20 mm× 3 mm. Prior to testing, the composite specimens were stored in a desiccator to avoid moisture absorption. For statistical purpose, a total of five samples for each tests were carried out at room temperature and the average values were recorded.

Fractured surfaces of composite specimens were studied by using a scanning electron microscope (Model: Hitachi S-3000N), to investigate the morphology and interface between the fibers and matrix in the composite.

2.4 Weibull statistics

Generally, composite materials are anisotropic and, therefore, have different mechanical properties in different directions. In addition to this, they present varying strengths due to their internal structure, which means that there is no specific strength value to represent their mechanical behavior. This leads to the necessity of employing statistical analyses for their safe utilization in design and manufacturing. The strength distribution of the composite materials obeys Weibull distribution, normal distribution, logarithmic normal distribution, etc.

The Weibull distribution model has been widely used in recent years to describe the statistical behavior of the strength properties of many materials, such as metallic matrix composites, ceramic matrix composites and PMCs. Two popular forms of this distribution are two- and three-parameter Weibull distributions. The (cumulative) distribution function of the three-parameter Weibull distribution is given as follows [27]:

$$F(\sigma_i; a, \sigma_0, m) = 1\text{-}\exp\left(\text{-}\left(\frac{\sigma_i\text{-}a}{\sigma_0}\right)^m\right) \quad a \geq 0, \sigma_0 \geq 0, m \geq 0 \qquad (3)$$

where a, σ_0, and m are the location, scale and shape parameters, respectively. When a=0 in Eq. (3), the distribution function of the two-parameter Weibull distribution is obtained. The three-parameter Weibull distribution is suitable for situations in which an extreme value cannot take values $<a$.

In the present communication, the tensile strength studies were carried out using the two-parameter Weibull distribution. This study uses the tensile strength test data; the research hypothesis assumed that the data obey two-parameter Weibull distribution. The maximum normed residual (*MNR*) method was applied to eliminate abnormal data and the KS test was used to examine the assumed distribution. The distribution function in this case can then be written as follows:

$$P = F(\sigma_i; \sigma_0, m) = 1\text{-}\exp\left(\text{-}\left(\frac{\sigma_i}{\sigma_0}\right)^m\right) \qquad (4)$$

where F is the probability of rupture of the material under uniaxial tensile stress σ_i, m is the shape parameter or Weibull modulus, and σ_0 is the scale parameter of the distribution. Weibull modulus, m, is related to the scatter of the data: the higher the m, the smaller the strength dispersion, and then the material is more uniform. It becomes the most important parameter of the distribution. The scale parameter is closely related to the mean strength.

3 Results and discussion

3.1 Density and void fraction

Density of a composite depends on the relative proportion of matrix and reinforcing materials and this is one of the most important factors determining the properties of the composites. The densities of the RV composite samples are shown in Figure 2. The density of composites decreased with increase in weight fraction of roselle fibers. This is due to the lower density of the roselle fiber than that of the vinyl ester matrix, thereby the resulting composite density obviously decreased. With addition of roselle fibers in vinyl ester resin matrix, the volume fraction of voids increased, as shown in Figure 3. The voids significantly affect some mechanical properties and even the performance of the composites in the place of use. The knowledge about the void content in composites was desirable for estimation of the quality of the composite materials. It was understandable that a good composite should have fewer voids. However, the presence of voids was unavoidable in composite-making, particularly through the hand lay-up technique [28].

3.2 Tensile properties

Many types of characterization tools, such as physical, chemical, mechanical, morphological and thermal properties, are available to determine the overall behavior of fiber-reinforced polymer composites. Among these tools, mechanical characterization is particularly one of the most significant tools in determining the properties of fiber-reinforced polymer composites under various load conditions. Generally, the properties of fiber-reinforced polymer composites are controlled by strength, distribution, and orientation of the reinforcing material, the nature of the resin matrix, and the properties of the fiber-matrix adhesion/bonding. In the case of the plant-based lignocellulosic natural fiber reinforced polymer composites, mechanical properties of resin matrix were found to increase when reinforced with lignocellulosic natural fiber materials.

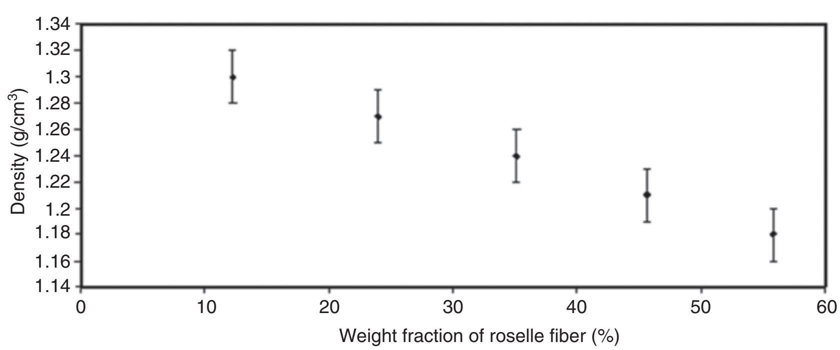

Figure 2: Variation of density with different weight percent of roselle fiber reinforced in vinyl ester resin matrix. [Fixed value: 0.02, percentage: 5, standard deviation (s): 1].

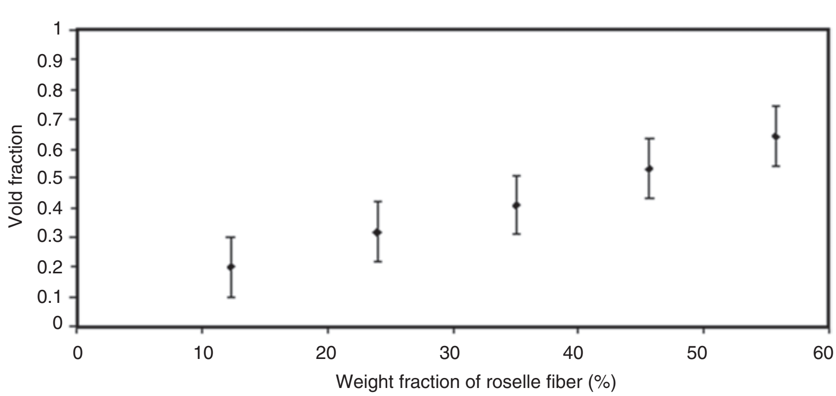

Figure 3: Variation of void fraction of composite with wt% of roselle fiber-reinforced vinyl ester composites. [Fixed value: 0.1, percentage: 5, standard deviation (s): 1].

Table 2 shows the tensile properties of neat resin samples and RV composites. Based on the results in Table 2, it is obvious that mechanical properties of vinyl ester composites increase upon reinforcement with roselle fibers. The reinforcement with roselle fibers has a significant effect on the tensile strength and modulus of composites. The tensile strength and tensile modulus values increased linearly with increasing fiber loading. The tensile strength and modulus of composite specimens increased with increasing fiber weight percentage until the fiber loading reached 45.67 wt%. Composites with 45.67 wt% fiber reinforcement produced the largest tensile strength value (40.3 MPa), which corresponds to a 38.5% improvement compared to the neat resin sample. At 55.77 wt% fiber, the tensile strength was slightly lower, which may be due to poor interfacial adhesion between the fibers and matrix. A loss of ductility in RV composites may be identified by increasing the roselle fiber loading, but no significant change in the tensile modulus was observed. This may be due to a poor fiber-matrix interfacial adhesion [29, 30]. Mechanical properties, particularly the tensile properties of natural fiber-reinforced polymers (both thermoplastics and thermosets), are mainly influenced by the interfacial adhesion between the matrix and the fibers. The tensile strengths and modulus of the plant-based lignocellulosic natural fiber reinforced polymer composites increased with weight or volume fraction of fiber, up to a maximum or optimum value, then the value will then drop [31]. The same values of tensile strength were identified at 35.08 wt% (36.8 MPa) and 55.77 wt% (36.5 MPa) fiber loading, respectively. At 55.77 wt% fiber, the maximum tensile modulus value was obtained. The tensile modulus showed an increasing trend from 12.29 wt% to 55.77 wt%. The highest tensile modulus value was achieved in composites having 55.77 wt%. The specific tensile strength and modulus of composites also increased linearly with weight fraction of fiber, as shown in Table 2.

Table 2: Average tensile properties of reinforced vinyl ester (RV) composite.

Fiber length (mm)	Fiber content (wt%)	Tensile strength (MPa)	Specific tensile strength (MPa)	Tensile modulus (MPa)	Specific tensile modulus (MPa)
0	0	29.1	25.282	1097.4	953.432
6	12.29	28.7	24.322	962.1	815.339
	23.97	30.1	24.876	1001.69	827.843
	35.08	36.8	29.677	1125.03	907.2823
	45.67	40.3	31.732	1193.2	939.5276
	55.77	36.5	28.077	1224.3	941.7692

As the weight fraction of fibers increased in composites, the specific tensile strength of composites with 35.08 wt% fibers (29.677 MPa) was higher than those of composites with 55.77 wt% fibers (28.077 MPa). The same specific tensile strength values were observed at 12.29 wt% and 23.97 wt%, respectively. According to Ku et al. [32] and Ahmad et al. [33], the high fiber loading is required to achieve high performance of the composites. Therefore, the effect of fiber loading on the properties of natural fiber-reinforced composites is particularly significant. It was often observed that the increase in fiber loading leads to an increase in tensile properties. The presence of fiber or other reinforcement in the polymeric matrix raises the composite strength and modulus [34].

3.3 Fractographic analysis

The fractured surface of the composite with 45.67 wt% fiber is shown in a scanning electron microscopy image in Figure 4A and B. From this image, we can observe poor interfacial adhesion between fibers and resin matrix. This was confirmed by fiber pull out and matrix disturbance. In order to achieve good fiber reinforcement, the interfacial strength between the fiber and matrix is the most essential factor. For a composite to be an effective load-bearing system, the fibers and matrix must cooperate. This cooperation between the fibers and the matrix will not exist without the presence of the interface. Figure 4C and D show the fractured surfaces of the composite specimens with fiber loading of 45.67 wt% with roselle fibers. Fiber fracture and less fiber pull out were seen on the fractured surface of the composite specimens. A significant amount of interfacial adhesion between fibers and matrix and also brittle mode of failure is seen. The improved adhesion in composites was evident from the broken fibers. Composite specimens which obtained better properties showed better interfacial adhesion.

3.4 Results of two-parameter Weibull distribution

From the experimental results of Table 1, the maximum tensile strength value was identified and then the corresponding fiber loading was taken for further processing. Composite specimens were fabricated and cut by using the above methods. The tensile tests were carried out according to the same ASTM standard, and with the same machine and test conditions. Twenty-five tension tests were performed and using the test data, the

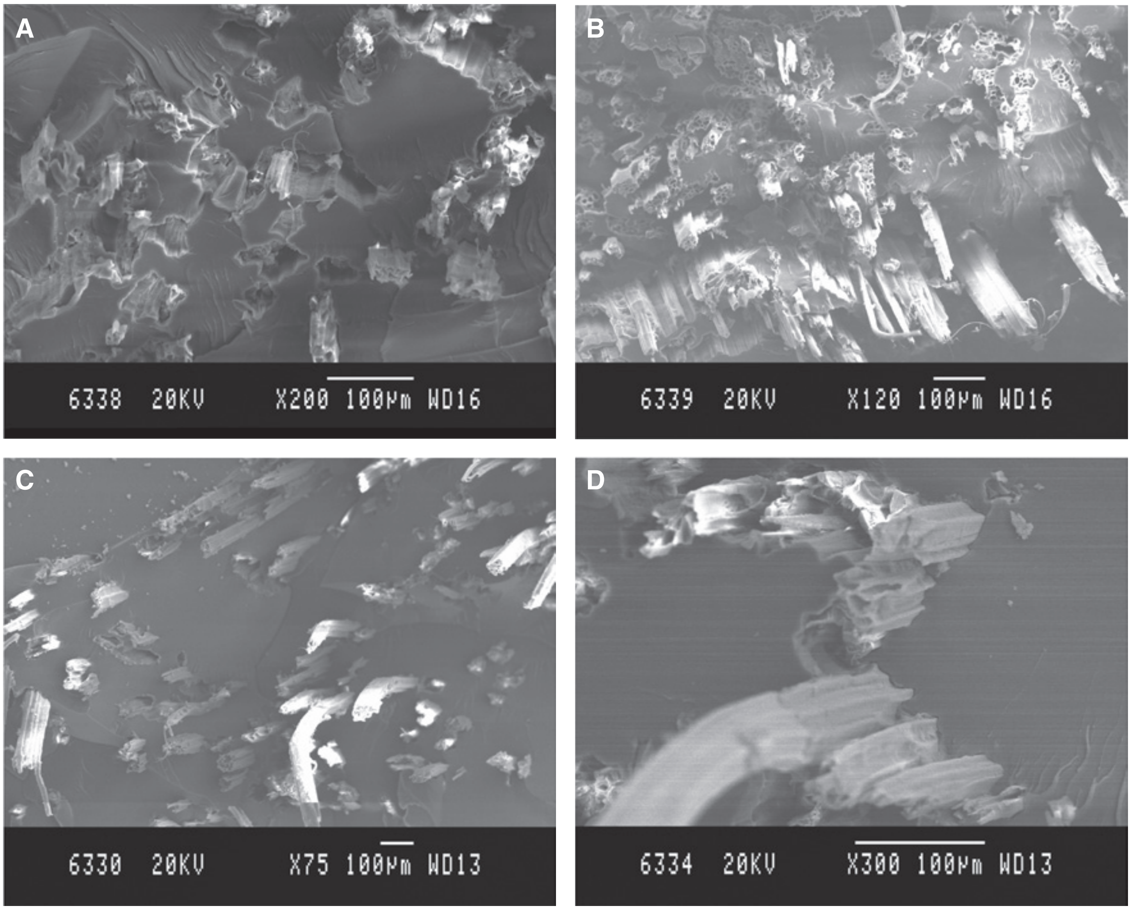

Figure 4: Scanning electron microscopy (SEM) image of fractured surface of tensile tested composite specimens: (A) experiment number=19, (B) experiment number=13, (C) experiment number=5 and (D) experiment number=16.

corresponding two-parameter Weibull distribution was determined.

After collection of test data from the laboratory, a visual inspection method was used to identify the abnormal data during analysis of the collected data. Abnormal data will be much higher or lower than the majority of the observed values. This may be due to several reasons, such as recording errors, using specimens with defects, or wrong environmental conditions. These abnormal data have a substantial impact on statistical analysis, so we decided to eliminate the abnormal data before statistical analysis. To eliminate the abnormal data, *MNR* is used. A comparison is carried out between the *MNR* value and the critical value when examining abnormal data value. If the *MNR* value is higher than the critical value *C*, there is abnormal data in the set of collected data. The identified abnormal data must be eliminated or corrected, and then the *MNR* method is used to inspect the data until there is no abnormal data. The *MNR* and critical values may be calculated by Eqs. (5) and (6).

$$MNR=\frac{\max|\sigma_i\text{-}\bar{\sigma}|}{S} \quad i=1, 2, \ldots, n \tag{5}$$

$$C=\frac{n\text{-}1}{\sqrt{n}}\sqrt{\frac{t^2}{n\text{-}2+t^2}} \tag{6}$$

where $\sigma_i=\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_n$ are the members of the data value, subscript i denotes the ith ($1\le i\le n$) member, $\bar{\sigma}$ is the sample mean, n is the size of the sample, S is the sample standard deviation and t is the t-distribution quantile as confidence is $[1\text{-}a/2n]$. The value of a is 0.05. The experimental tensile strength values and *MNR* values are shown in Table 3. According to the tensile strength test data (σ_i) of the composite, the mean ($\bar{\sigma}$) and the standard deviation (S) were 41.516 MPa and 1.971, respectively.

According to Table 2, when calculating results, *MNR* values are calculated on the basis of Eq. (5), when the sample degrees of freedom n=25, the critical value C=4.79 on the basis of Eq. (6), where t=93.93. Since the *MNR* value is less than the critical value C, there are no abnormal data

Table 3: Experimental tensile strength values and corresponding maximum normed residual (*MNR*) values.

Experiment no.	Tensile strength (σ_i) (MPa)	*MNR*
1	43.32	0.915
2	41.28	0.119
3	39.64	0.952
4	44.45	1.489
5	41.56	0.022
6	43.89	1.205
7	42.24	0.367
8	41.75	0.119
9	43.92	1.219
10	37.89	1.840
11	38.73	1.414
12	43.32	0.915
13	39.84	0.851
14	43.21	0.859
15	38.99	1.282
16	39.11	1.221
17	42.87	0.687
18	40.67	0.429
19	39.44	1.053
20	40.33	0.602
21	41.47	0.023
22	45.07	1.803
23	41.44	0.039
24	40.56	0.485
25	42.91	0.707

in the set of collected data. Suppose, if there were abnormal data, the values were corrected and the corresponding mean and the standard deviation of the corrected data were recalculated.

In the next step, the parameters of two-parameter Weibull distribution were estimated. The shape parameter (m) or Weibull modulus and the scale parameter (σ_0) are calculated to inspect the assumed two-parameter Weibull distribution. The parameters σ_0 and m of the Weibull distribution function F (σ_i; σ_0, m) are estimated from observations. The methods usually employed in the estimation of these parameters are the method of linear regression, the method of maximum likelihood and the method of moments with the help of Weibull graphs [35]. However, software programs with statistical abilities such as MS Excel, SPSS and Microcal Origin can be used to replace the Weibull graph papers. So, the parameter estimation is done first. In order to compute m and σ_0, they are first ordered from the smallest to the largest and computed. The test specimen probability fracture is obtained from Eq. (7):

$$P=F(\sigma_i)=\frac{i\text{-}0.5}{n} \tag{7}$$

where n is the time of the total test and i is the ith time of the test. The transformed results after taking the logarithm on both sides of the Eq. (4) were:

$$Y_i=\ln\left(\ln\left[\frac{1}{(1\text{-}P)}\right]\right) \tag{8}$$

$$X_i=\ln(\sigma_i) \tag{9}$$

$$B=\text{-}m\ln(\sigma_0) \tag{10}$$

$$Y_i=mX_i+B. \tag{11}$$

In Eq. (4), $F(\sigma_i;\ \sigma_0,\ m)$ represents the probability that the tensile strength is $\leq\sigma_i$. Using the equality $F(\sigma_i;\ \sigma_0,\ m)+R(\sigma_i;\ \sigma_0,\ m)=1$, the reliability $R(\sigma_i;\ \sigma_0,\ m)$, that is, the probability that the tensile strength is at least σ_i, is defined as [35]:

$$R(\sigma_i;\sigma_0,m)=\exp\left(\text{-}\left(\frac{\sigma_i}{\sigma_0}\right)^m\right)\quad \sigma_0\geq 0,m\geq 0. \tag{12}$$

The two-parameter Weibull distribution function is transformed into a linear relationship as shown in Eq. (11). Draw the linear line between X_i and Y_i and the plot is known as the Weibull probability plot (WPP) of the composite. If the regression line is a straight line, the correlation coefficient of X_i and Y_i is close to 1, and the composite material strength distribution can use the two-parameter Weibull distribution to be described, otherwise, it is not suitable for the two-parameter Weibull distribution. It was convenient to get the parameters of the two-parameter Weibull distribution by the parameters m and B of the fitting line. Figure 5 shows the WPP of the composite material. Table 4 shows the experimental tensile strength values and corresponding Y_i and X_i values.

The slope of the line is 27.229, which is the value of the shape parameter m. A values of m<1:0 indicates that the material has a decreasing failure rate. Similarly, m=0 indicates constant failure rate and m>1:0 indicates an increasing failure rate. The σ_0 value is computed as σ_0=42.34 using the point the line intersects the Y axis (-101.93) in $\sigma_0=\exp[\text{-}(y/m)]$. Therefore, m=27.229 indicates that the composite material tends to fracture with a higher probability for every unit increase in applied tension load. The scale parameter σ_0 measures the spread in the distribution of data. Then the distribution function was:

$$P=F(\sigma_i;\sigma_0,m)$$
$$=\int_0^\sigma m\sigma_0^{\text{-}m}t^{m\text{-}1}\exp\left(\text{-}\left(\frac{t}{\sigma_0}\right)^m\right)dt=1\text{-}\exp\left(\text{-}\left(\frac{\sigma}{42.34}\right)^{27.229}\right). \tag{13}$$

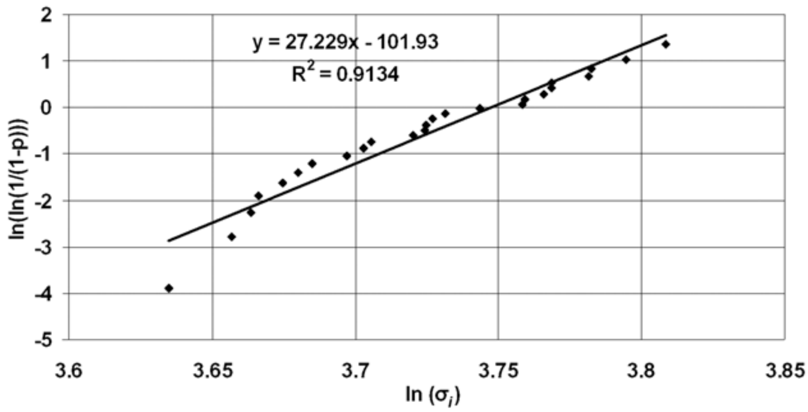

Figure 5: Weibull probability plot of reinforced vinyl ester (RV) composites.

3.5 Description of tensile strength in terms of reliability

From Eq. (12), $R(\sigma_0; \sigma_0, m)=0.509$. Therefore, $R(42.34; 42.34; 27.229)=\exp(-(\sigma_i/\sigma_0)^m=0.509$, that is 50.9% of the tested RV composite specimens have a tensile strength of at least 42.34 MPa. Figure 6 shows the reliability $R(\sigma_i; \sigma_0, m)$ plot.

Table 4: Experimental tensile strength values and corresponding Y_i and X_i values.

Experiment no.	Tensile strength (σ_i) (MPa)	P	Y_i	X_i
1	37.89	0.02	3.90	3.63
2	38.73	0.06	2.78	3.66
3	38.99	0.1	2.25	3.66
4	39.11	0.14	1.89	3.67
5	39.44	0.18	1.62	3.67
6	39.64	0.22	1.39	3.68
7	39.84	0.26	1.20	3.68
8	40.33	0.3	1.03	3.69
9	40.56	0.34	0.88	3.70
10	40.67	0.38	0.74	3.71
11	41.28	0.42	0.61	3.72
12	41.44	0.46	0.48	3.72
13	41.47	0.5	0.37	3.72
14	41.56	0.54	0.25	3.73
15	41.75	0.58	0.14	3.73
16	42.24	0.62	0.03	3.74
17	42.87	0.66	0.08	3.76
18	42.91	0.7	0.19	3.76
19	43.21	0.74	0.29	3.77
20	43.32	0.78	0.41	3.77
21	43.32	0.82	0.54	3.77
22	43.89	0.86	0.68	3.78
23	43.92	0.9	0.83	3.78
24	44.45	0.94	1.03	3.79
25	45.07	0.98	1.36	3.81

It was observed that tensile strength values approximately ≤37.89 MPa will provide high reliability. For a more certain assessment, consider 0.90 and 0.95 reliability levels. When these values are put as $R(\sigma_i; \sigma_0, m)$ in Eq. (12) and the equation is solved for σ_i, the tensile strength values 38.98 MPa and 37.96 MPa are obtained, respectively. In other words, this RV composite material will fail with 0.90 probability for a tension of 38.98 MPa or more, and similarly will fail with 0.95 probability for a tension of 37.96 MPa or more.

3.6 Inspection of two-parameter Weibull distribution using the KS method

The KS method is used to inspect the hypothesis distribution of the tensile strength of the composites. KS is based on the cumulative distribution function, comparing the cumulative probability gap between the theoretical frequency and actual frequency, to find the maximum distance D. Whether the actual frequency distribution is

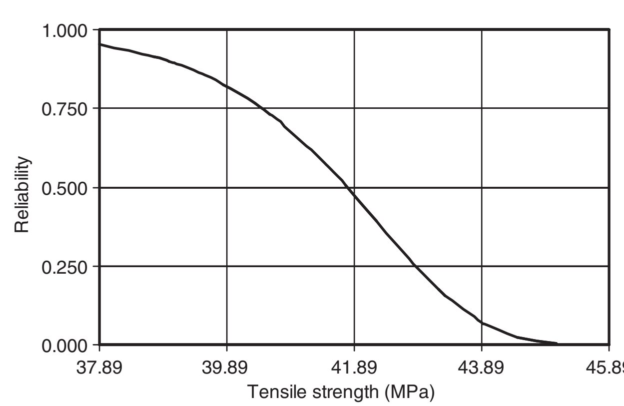

Figure 6: Reliability $R(\sigma_i; \sigma_0, m)$ plot.

amenable to the theoretical frequency distribution or not is determined by the value of D. The maximum distance D can be calculated by Eq. (14):

$$D=\max[|F(\sigma_i)-G(\sigma_i)|] \tag{14}$$

where $F(\sigma_i)$ is the empirical distribution function and $G(\sigma_i)$ is the theoretical distribution function. $\sigma_1, \sigma_2, \ldots, \sigma_n$ is a set of samples from $F(\sigma_i)$, and supposes $F(\sigma_i)$ is amenable to the theoretical distribution, that is to inspection $[H_0{:}F(\sigma_i)=G(\sigma_i),\ H_1{:}F(\sigma_i)\neq G(\sigma_i)]$. Under the given significance level, the critical value $D_{n,\alpha}$ can be looked up when $P(D>D_{n,\alpha})=\alpha$. If the maximum distance $D<D_{n,\alpha}$, the null hypothesis is accepted, that is $H_0{:}F(\sigma_i)=G(\sigma_i)$. Otherwise, the null hypothesis is refused. Table 5 shows the inspection process of the two-parameter Weibull distribution.

The Kolmogorov test statistic $D=0.0958$, taking $\alpha=0.05$, the critical value $D_{n,\alpha}$ can be looked up from the critical value table, i.e., $D_{25,0.05}=0.26404>D=0.0958$. So, the null hypothesis is accepted according to the KS method, and the tensile strength of composite material can use the inspected two-parameter Weibull distribution to be described. From the result of this inspection, we know that the tensile strength distribution of the composite materials can be displayed by the two-parameter Weibull distribution.

Table 5: Inspection process of two-parameter Weibull distribution.

Experiment no.	Tensile strength (σ_i) (MPa)	Probability of experimental tensile strength (P)	Probability of Weibull tensile strength (P_0)	$\lvert P-P_0\rvert$
1	37.89	0.020	0.047	0.03
2	38.73	0.060	0.085	0.02
3	38.99	0.100	0.101	0.00
4	39.11	0.140	0.109	0.03
5	39.44	0.180	0.135	0.05
6	39.64	0.220	0.153	0.07
7	39.84	0.260	0.174	0.09
8	40.33	0.300	0.234	0.07
9	40.56	0.340	0.267	0.07
10	40.67	0.380	0.284	0.10
11	41.28	0.420	0.394	0.03
12	41.44	0.460	0.427	0.03
13	41.47	0.500	0.433	0.07
14	41.56	0.540	0.453	0.09
15	41.75	0.580	0.495	0.09
16	42.24	0.620	0.608	0.01
17	42.87	0.660	0.754	0.09
18	42.91	0.700	0.763	0.06
19	43.21	0.740	0.824	0.08
20	43.32	0.780	0.845	0.07
21	43.32	0.820	0.845	0.03
22	43.89	0.860	0.930	0.07
23	43.92	0.900	0.934	0.03
24	44.45	0.940	0.977	0.04
25	45.07	0.980	0.996	0.02

3.7 Prediction of average tensile strength

The average tensile strength value can be calculated using the following equations. Weibull distribution is shown below on the basis of Eq. (4):

$$f(\sigma_i)=\frac{dF(\sigma_i)}{d\sigma_i}=\frac{m}{\sigma_0}\times\left(\frac{\sigma_i}{\sigma_0}\right)^{m-1}\times\exp\left(-\left(\frac{\sigma_i}{\sigma_0}\right)^m\right) \tag{15}$$

The mean value of the tensile strength is:

$$\overline{\sigma_i}=\int_0^\infty \sigma_i f(\sigma_i)\times d\sigma_i=\int_0^\infty \sigma_i\frac{m}{\sigma_0}\times\left(\frac{\sigma_i}{\sigma_0}\right)^{m-1}\times\exp\left(-\left(\frac{\sigma_i}{\sigma_0}\right)^m\right)\times d\sigma. \tag{16}$$

To calculate the mean of the tensile strength, substitute the value of the parameters m and σ_0 into Eq. (16); we get a mean value of the tensile strength of 41.40 MPa. The measured average value was 41.52 MPa. The deviation was only 0.29%. The precision was high from the perspective of a reliability design, so using the two-parameter Weibull distribution to express the tensile strength distribution was reasonable.

4 Conclusions

The tensile properties of RV composites were evaluated based on the five different fiber loadings (12.29 wt%, 23.97 wt%, 35.08 wt%, 45.67 wt% and 55.77 wt%) and constant fiber length (6 mm). The density of composites decreased with increase in loading of roselle fibers. The volume fraction of voids increased with addition of roselle fibers in vinyl ester resin matrix. The tensile property (tensile strength) values increased linearly with increasing fiber weight percentage. The tensile strengths of the RV composites increased with fiber loading up to 45.67 wt% and then dropped. Specific tensile strengths of composites with 35.08 wt% fibers (29.677 MPa) were higher than those of composites with 55.77 wt% fibers (28.077 MPa). Both the tensile modulus and specific tensile modulus reached the highest values at composites with 55.77 wt% of fibers. For statistical analysis, 25 experiments were carried out on composite specimens with the same fiber loading (45.67 wt%), fiber length (6 mm) and size (150×20×3 mm) and corresponding tensile strength values were recorded. The parameters (m and σ_0) of the two-parameter Weibull distribution as a model were calculated using a graphic method. Finally, the KS method was used to inspect the distribution hypothesis and predict the mean value. The deviation was very small (0.29%) when comparing the predictive value and the measured average. Therefore, the tensile strength of RV composites can be expressed by the two-parameter Weibull distribution.

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