# Dynamic Characteristics of Twisted Composite Panels

**V.Hariharan, G.Rajeshkumar, P.Thangavel**

***Abstract: Composite materials are being increasingly used in turbo-machinery blades because of their higher specific strength and stiffness, and these can be tailored through the variation of fiber orientation and stacking sequence to obtain an efficient design. Thus the dynamic characteristics of laminated composite twisted cantilever panels are of great importance in engineering applications. Analytical solutions are seldom capable of accurately analyzing these types of problems because of the complex orthographic nature of material and geometry. The finite element method (FEM) is the most commonly used numerical technique. The present research work deals with the review of finite element formulation of dynamic problems applied to orthotropic laminated plates and to carry out a parametric FEM analysis of laminated composite twisted curved panels. The effects of parameters like twist angle, stacking sequence and thickness, lamination angle, and orthotropic ratio on the natural frequencies and mode shapes of twisted cantilevered composite plates are investigated.***

***Key Words: Composite material, twisted composite beam, Natural frequencies, Mode shapes.***

## I. INTRODUCTION

Composite materials have emerged as an advanced class of structural materials for aircraft, spacecraft, automotive, marine and allied industries. The twisted composite cantilever panels have significant applications in wide-chord turbine blades, compressor blades, fan blades and particularly in gas turbines. The range of practical applications demands a proper understanding of their vibration, static and dynamic stability characteristics. Due to its significance, a large number of references deal with the free vibration of twisted plates. Structural elements subjected to in-plane periodic forces may lead to parametric resonance, due to certain combinations of the values of load parameters. The instability may occur below the critical load of the structure under compressive loads over wide range of excitation frequencies.

Thus the parametric resonance characteristics of laminated composite twisted cantilever panels are of great importance for understanding the systems under periodic loads. The dynamic stability behavior of a rotating blade subjected to axial periodic forces is studied by Lagrange's equation and a Galerkin finite element method by Chen and Peng (1995). The effects of geometric non-linearity, shear deformation and rotary inertia are considered. The iterative method is used to get the mode shapes and frequencies of the non-linear system. Dynamic instability regions of the blade with different reference amplitudes of vibration are illustrated graphically. Ganapathi et al. (1994) investigates the complex boundary conditions and different type of loads where analytical methods are not easily applicable. Finite elements based on a field consistency principle by Isaac et al. (1994) have been developed recently for the structural analysis of thick as well as thin plates/shells by Lim and Kitipornchai (2000). An excellent survey of the earlier works in the free vibration of turbo-machinery blades was carried out by Leissa (1981).

The majority of earlier researchers treated the blades as beams that are one-dimensional case. Such an idealization is highly inaccurate for the blade with moderate to low aspect ratio. Leissa et al. (1982) employed shallow shell theory and Ritz method to determine the frequencies of turbo-machinery blades with twist for different degrees of Shallowness and considered involves the estimation of frequencies and corresponding mode shape in a simply supported square plate. The results obtained were compared with close form solution to ensure the accuracy of finite element method. Figure-1 shows the geometry and dimensions of the square plate thickness. Lien-Wen Chen and Jenq-Ying Yang (1990) in his work investigated the dynamic stability of laminated composite plates due to periodic in-plane loads. Qatu and Leissa (1990) determine the free vibration of laminated composite twisted cantilever plates using Ritz method. In his work the effect of the thickness ratio, angle of twist and fiber orientation angle upon the natural frequencies and mode shapes of three-layer-glass/epoxy and graphite/epoxy angle-ply plates were studied. This work presents the first known natural frequencies and mode shapes of laminated cantilever plates having pre-twist. Laminated shallow-shell theory and Ritz method are used in this work

## II. GEOMETRY AND DIMENSIONS OF SQUARE PLATE

For the purposes of verifying the twisted model a sample rectangle model plate of thickness t is first considered for analysis. The problem

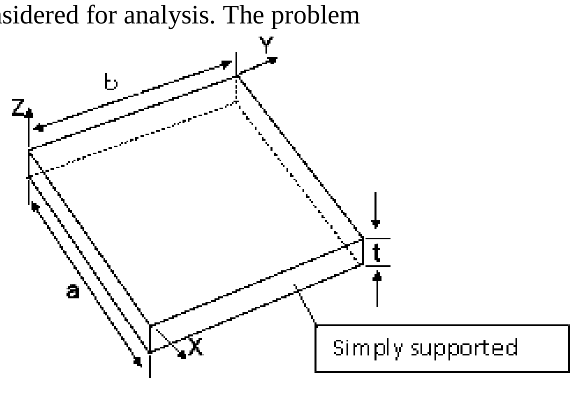


**Figure 1. Geometry of square plate**

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**Dr.V.Hariharan**, Professor of Mechanical Engineering at Kongu Engineering College, Tamilnadu, India.

**Dr.G.Rajeshkumar** , Associate Professor of Mechanical Engineering at PSG Institute of Technology and Applied Research, India, Tamilnadu India.

**Dr.P.Thangavel**, Associate Professor of Mechanical Engineering at Kongu Engineering ,Tamilnadu, India.

a-Width of the plate = 0.1 m; b-Height of the plate = 0.1 m; t-Thickness of the plate = 0.001 m

### A Material Properties

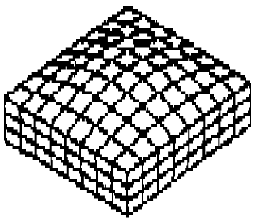
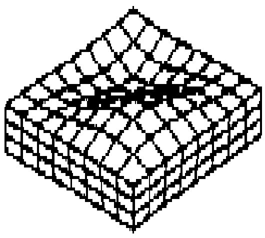
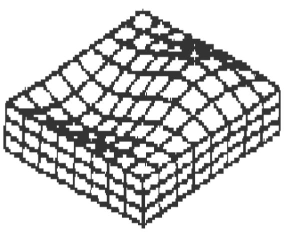
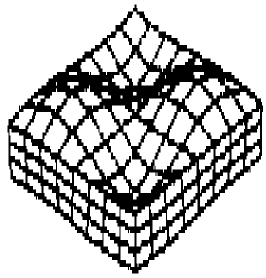
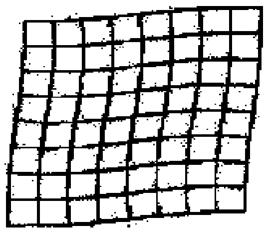
The material properties considered in the present study are:

Material = Steel
Modulus of elasticity $E = 200 \times 10^9$ N/m$^2$
Poisson Ratio $\mu = 0.3$
Density $\rho = 8000$ kg/m$^3$

### B Finite Element Solution

The details of finite element modeling and analysis are discussed in this section. In ANSYS under pre-processing stage the elements are used for analysis is described depending upon the accuracy requirements. In this present work Solid 45 element is selected for analyzing the square plate. In general the Solid 45 element is used for 3D modeling of solid structures. The element is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. The element has properties like plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities. Figure 2 shows the geometry of the Solid 45 element and Figure 3 shows the boundary condition of the meshed model. All the side of the square plate is simply supported as shown in Figure 3. The number of elements considered for the analysis is limited to 3200, because further increase in numbers of elements does not have any significant effect on natural frequencies and mode shapes. The vibration analysis is carried out to determine its natural frequencies and mode shapes that are plotted and given in Table-1.

**Table 1. FEM result of steel plate**

| Mode Number | Frequencies (Hz) | Mode shapes |
|---|---|---|
| 1 | 45.897 |  |
| 2 | 109.44 |  |
| 3 | 109.44 |  |
| 4 | 167.89 |  |
| 5 | 193.59 |  |

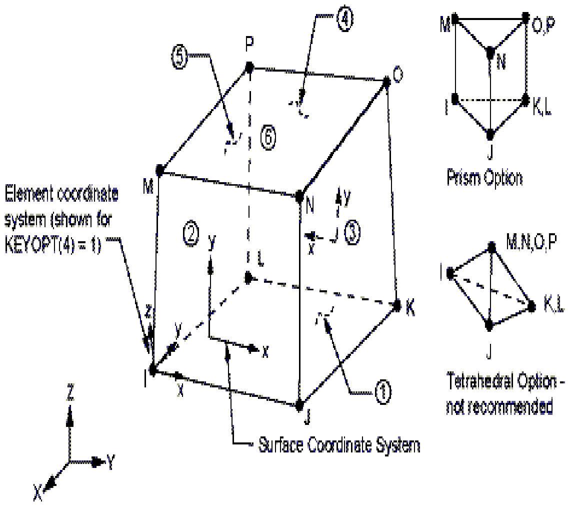



**Figure 2. Solid 45 element geometry**

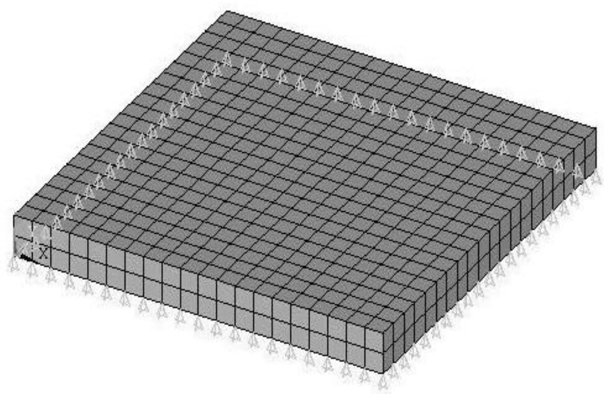


**Figure 3. FEM mesh and boundary condition**

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**C Analytical Method**

The general class of laminated square plate that are simply supported along edges x = 0, x = a, y = 0 and y = b as shown in Figure 1. More complicated boundary conditions and the effect of an equilibrium stress state are considered. The vibration frequencies and modes are described by a single vibration differential equation. (Robert, 1999)

$D_{11}\,\delta w,_{xxxx} + 2\,(D_{12} + 2D_{66})\,\delta w,_{xxyy} + D_{22}\,\delta w,_{yyyy} + \rho\,\delta w,_{tt} = 0$ (1)

Subject to the simply supported edge boundary conditions

$x = 0, a$: $\delta w = 0$ $\delta M_x = - D_{11}\ \delta w,_{xx} - D12\ \delta w,_{yy} = 0$

$y = 0, b$: $\delta w = 0$ $\delta M_y = - D_{12}\ \delta w,_{xx} - D22\ \delta w,_{yy} = 0$

(2)

The harmonic solution is given by

$\delta w\,(x,y,t) = (A \cos \omega t + B \sin \omega t)\,\delta w\,(x,y)$ (3)

by considering time and spatial variations from Equation 3.

$\delta w\,(x,y) = \sin(m\pi x/a)\sin(n\pi y/b)$

$\omega^2 = (\pi^4 / \rho)\,\{D_{11}\,(m/a)^4 + 2(D_{12} + 2D_{66})\,(m/a)^2\,(n/b)^2 + D_{22}\,(n/b)^4\}$ (4)

Frequency $f = \omega / 2\pi$

Various frequencies of ω, correspond to different mode shapes by changing the different values of m and n in Equation 4.

For a single isotropic plate

$D_{11} = Et^3 / 12(1-\mu^2) = D$

$D_{12} = \mu D$

$D_{11} = D_{22} = D$

$D_{16} = D_{26} = 0$

$D_{66} = (1-\mu)\,D/2$

Determining the first frequency and corresponding mode shape: applying m=1, n=1 in Equation 4.

$\omega^2 = (\pi^4 / \rho)\,\{D_{11}\,(m/a)^4 + 2(D_{12} + 2\,D_{66})\,(m/a)^2\,(n/b)^2 + D_{22}\,(n/b)^4\}$

$\omega^2 = (\pi^4 / \rho)\,\{D_{11}\,(1/a)^4 + 2(D_{12} + 2\,D_{66})\,(1/a)^2\,(1/b)^2 + D_{22}\,(1/b)^4\}$

Apply a = b = 10

$\omega^2 = (\pi^4 / \rho)\,\{D_{11}\,(1/10)^4 + 2(D_{12} + 2D_{66})\,(1/10)^2\,(1/10)^2 + D_{22}\,(1/10)^4\}$

$\omega^2 = (\pi^4 / \rho \times 10^4)\,\{D_{11} + 2(D_{12} + 2D_{66}) + D_{22}\}$

Apply $D_{11} = D22 = D$; $D_{12} = \mu D$; $D_{66} = (1-\mu)\,D/2$

$\omega^2 = (\pi^4 / \rho \times 10^4)\,\{D + 2(\mu D + 2(1-\mu)\,D/2) + D\}$

$\omega^2 = (\pi^4 / \rho \times 10^4)\,\{D + 2\mu D + 2D - 2\mu D + D\}$

$\omega^2 = (\pi^4 / \rho \times 10^4)\,\{D + 2D + D\}$

$\omega^2 = 4D\pi^4 / \rho \times 10^4$

$\boldsymbol{\omega = 2\pi^2 / 10^2\,(D/\rho)^{0.5}}$ apply value of ρ, D

$\omega = 2\pi^2 / 10^2\,(1.8315 \times 10^{10} / 8000)^{0.5} = 298.6$

$f = 298.6 / 2\pi$

**f = 47.5 Hz**

Determining the second frequency and corresponding mode shape: applying m=1, n=2 in Equation 4

$\omega^2 = (\pi^4 / \rho)\,\{D_{11}\,(m/a)^4 + 2(D_{12} + 2\,D_{66})\,(m/a)^2\,(n/b)^2 + D_{22}\,(n/b)^4\}$

$\omega^2 = (\pi^4 / \rho)\,\{D_{11}\,(1/a)^4 + 2(D_{12} + 2\,D_{66})\,(1/a)^2\,(2/b)^2 + D_{22}\,(2/b)^4\}$

Apply a = b = 10

$\omega^2 = (\pi^4 / \rho)\,\{D_{11}\,(1/10)^4 + 2(D_{12} + 2D_{66})\,(1/10)^2\,(2/10)^2 + D_{22}\,(2/10)^4\}$

$\omega^2 = (\pi^4 / \rho \times 10^4)\,\{D_{11} + 2(D_{12} + 2D_{66})8 + 16D_{22}\}$

Apply $D_{11} = D22 = D$; $D_{12} = \mu D$; $D_{66} = (1-\mu)\,D/2$

$\omega^2 = (\pi^4 / \rho \times 10^4)\,\{D + 2(\mu D + 2(1-\mu)\,D/2)\,4 + 16D\}$

$\omega^2 = (\pi^4 / \rho \times 10^4)\,\{D + (2\mu D + 2D - 2\mu D)\,4 + 16D\}$

$\omega^2 = (\pi^4 / \rho \times 10^4)\,\{D + 8D + 16D\}$

$\omega^2 = 25D\pi^4 / \rho \times 10^4$

$\boldsymbol{\omega = 5\pi^2 / 10^2\,(D/\rho)^{0.5}}$ apply value of ρ, D

$\omega = 5\pi^2 / 10^2\,(1.8315 \times 10^{10} / 8000)^{0.5} = 746.4$

$f = 746.4 / 2\pi$

**f = 118.8 Hz**

Determining the third frequency and corresponding mode shape: applying m=2, n=1 in Equation 4.

$\omega^2 = (\pi^4 / \rho)\,\{D_{11}\,(m/a)^4 + 2(D_{12} + 2\,D_{66})\,(m/a)^2\,(n/b)^2 + D_{22}\,(n/b)^4\}$

$\omega^2 = (\pi^4 / \rho)\,\{D_{11}\,(2/a)^4 + 2(D_{12} + 2\,D_{66})\,(2/a)^2\,(1/b)^2 + D_{22}\,(1/b)^4\}$

Apply a = b = 10

$\omega^2 = (\pi^4 / \rho)\,\{D_{11}\,(2/10)^4 + 2(D_{12} + 2D_{66})\,(1/10)^2\,(2/10)^2 + D_{22}\,(1/10)^4\}$

$\omega^2 = (\pi^4 / \rho \times 10^4)\,\{16D_{11} + 2(D_{12} + 2D_{66})8 + D_{22}\}$

Apply $D_{11} = D22 = D$; $D_{12} = \mu D$; $D_{66} = (1-\mu)\,D/2$

$\omega^2 = (\pi^4 / \rho \times 10^4)\,\{16D + 2(\mu D + 2(1-\mu)\,D/2)\,4 + D\}$

$\omega^2 = (\pi^4 / \rho \times 10^4)\,\{16D + (2\mu D + 2D - 2\mu D)\,4 + D\}$

$\omega^2 = (\pi^4 / \rho \times 10^4)\,\{16D + 8D + D\}$

$\omega^2 = 25D\pi^4 / \rho \times 10^4$

$\boldsymbol{\omega = 5\pi^2 / 10^2\,(D/\rho)^{0.5}}$ apply value of ρ, D

$\omega = 5\pi^2 / 10^2\,(1.8315 \times 10^{10} / 8000)^{0.5} = 746.4$

$f = 746.4 / 2\pi$

**f = 118.8 Hz**

Determining the fourth frequency and corresponding mode shape: applying m=2, n=2 in Equation 4.

$\omega^2 = (\pi^4 / \rho)\,\{D_{11}\,(m/a)^4 + 2(D_{12} + 2\,D_{66})\,(m/a)^2\,(n/b)^2 + D_{22}\,(n/b)^4\}$

$\omega^2 = (\pi^4 / \rho)\,\{D_{11}\,(2/a)^4 + 2(D_{12} + 2\,D_{66})\,(2/a)^2\,(2/b)^2 + D_{22}\,(2/b)^4\}$

Apply a = b = 10

$\omega^2 = (\pi^4 / \rho)\,\{D_{11}\,(2/10)^4 + 2(D_{12} + 2D_{66})\,(2/10)^2\,(2/10)^2 + D_{22}\,(2/10)^4\}$

$\omega^2 = (\pi^4 / \rho \times 10^4)\,\{16D_{11} + 2(D_{12} + 2D_{66})16 + 16D_{22}\}$

Apply $D_{11} = D22 = D$; $D_{12} = \mu D$; $D_{66} = (1-\mu)\,D/2$

$\omega^2 = (\pi^4 / \rho \times 10^4)\,\{16D + 2(\mu D + 2(1-\mu)\,D/2)\,16 + 16D\}$

$\omega^2 = (\pi^4 / \rho \times 10^4)\,\{16D + (2\mu D + 2D - 2\mu D)\,16 + 16D\}$

$\omega^2 = (\pi^4 / \rho \times 10^4)\,\{16D + 32D + 16D\}$

$\omega^2 = 64D\pi^4 / \rho \times 10^4$

$\boldsymbol{\omega = 8\pi^2 / 10^2\,(D/\rho)^{0.5}}$ apply value of ρ, D

$\omega = 8\pi^2 / 10^2\,(1.8315 \times 10^{10} / 8000)^{0.5} = 1194.6$

$f = 1194.6 / 2\pi$ = **190.1 Hz**

Table-2 shows the comparison of finite element model and analytical solutions of natural frequency and the mode shapes (m,n). The participation factor (k) is the sensitivity of the mode. It is a good indication of the importance of the state to the mode. Participation factors can be positive, zero, or negative. A positive participation factor associated with a particular generator means that generator is contributing to the oscillation of the system. A negative participation factor indicates a generator that is dampening the system oscillation. The participation factor data for this paper are all nonnegative and normalized.

Participation factors can be used to help determine whether a power system stabilizer (PSS) is needed to damp system vibrations. If the participation factors for many generators in an area are large, then a PSS placed in that area would dampen the oscillation of the system. However, if the participation factors for generators are negative, then adding a PSS would actually increase the oscillation

**Table 2. Comparison of FEM and Analytical Frequencies of steel plate**

| Mode Number | Analytical Frequencies (Hz) | FEM Frequencies (Hz) | Factor for mode shape (k) | m | n |
|---|---|---|---|---|---|
| 1 | 47.5 | 45.897 | 2 | 1 | 1 |
| 2 | 118.8 | 109.44 | 5 | 1 | 2 |
| 3 | 118.8 | 109.44 | 5 | 2 | 1 |
| 4 | 190.1 | 167.89 | 8 | 2 | 2 |
| 5 | 261.4 | 193.59 | 11 | 1 | 3 |

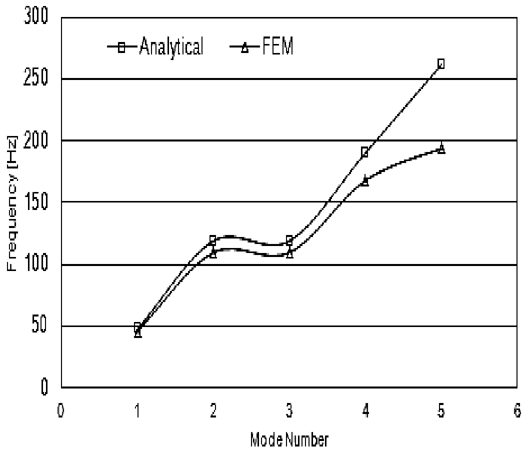


**Figure 4. Frequency comparison graph of steel plate**

From Figure-4 it is clear that the finite element method value is in close agreement with the analytical solution which establishes the capability of FEM solver code in the study of vibration analysis of twisted plate.

## III. DYNAMIC CHARACTERISTIC OF TWISTED COMPOSITE CURVED PANELS USING FEM

The investigation is carried out to evaluate the dynamic response of twisted composite laminate plates. The composite laminates considered in this present study are E-glass/epoxy and graphite/epoxy. E-glass/epoxy laminated plate with three ply symmetric arrangement [θ/-θ/θ] is used and three parametric variations considered are given below-

a) Figure 5 shows a laminate ply angle of [θ/-θ/θ]. It is varied from 0º to 90º in steps of 15º.
b) Figure 6 shows angle of twist Φ. Angle of twist varied from 0º to 45º in steps of 15º.
c) Two values of aspect ratio of the laminated plate (b/h) consider are b/h=20 and 100 is illustrated in Figure 7.

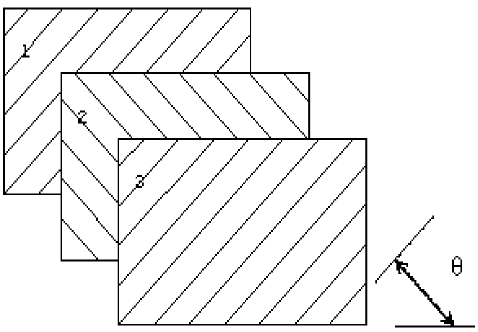


**Figure 5. Laminate Ply angle**

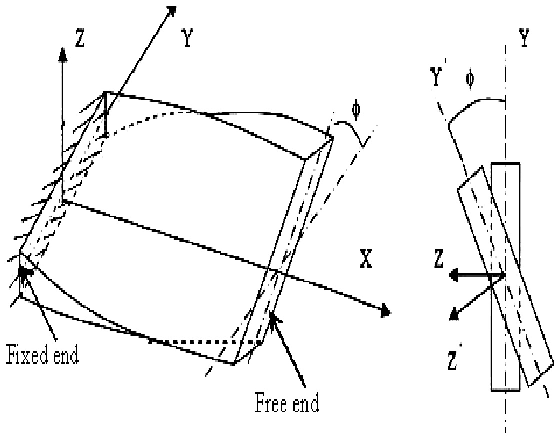


**Figure 6. Angle of twist**

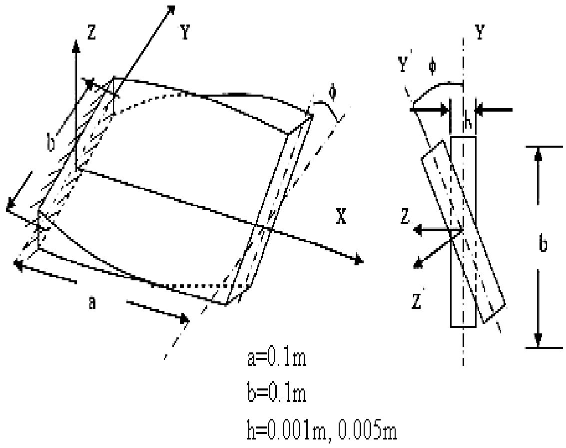


**Figure 7. Plate aspect ratio**

### A Finite Element Analysis of E-lass/epoxy laminates

After the composite plate has been modeled in the Ansys, meshing of the model is done which is shown in the Figure 8. In this study the laminate plate is assumed to be cantilever plate for which supporting boundary condition is applied as shown in the Figure 9.

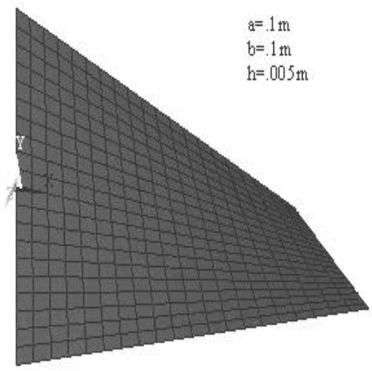


**Figure 8. FEM model of E-glass/epoxy**

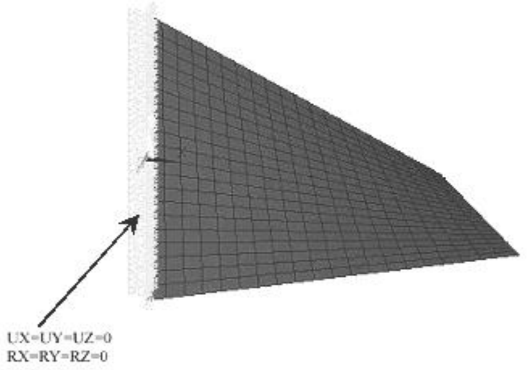


**Figure 9. Boundary condition of E-glass/epoxy**

The Material properties considered for the E-glass-epoxy laminated plates are shows in Table 3

**Table 3. Material properties for E-glass/epoxy**

| Material property | Value |
|---|---|
| Modulus of Elasticity | |
| $E_{XX}$ | $60x10^9$ N/m$^2$ |
| $E_{YY}$ | $24.8x10^9$ N/m$^2$ |
| $E_{ZZ}$ | $24.8x10^9$ N/m$^2$ |
| Modulus of Rigidity | |
| $G_{XY}$ | $12x10^9$ N/m$^2$ |
| $G_{YZ}$ | $12x10^9$ N/m$^2$ |
| $G_{XZ}$ | $12x10^9$ N/m$^2$ |
| Poisson ratio μ | 0.3 |
| Density ρ | 2324.76 kg/m$^3$ |
| Volume fraction | |
| E-glass | 83.26% |
| Epoxy | 16.73% |

### B. Results and discussion

First three Eigen values and corresponding modal shapes were extracted for all the case studied. The results of the investigation of frequencies are presented in Table 4 to 11. by varying orientation angles from 0° to 45° of twist and two aspect ratio 20 and 100. The table also incorporates the results of analytical investigations obtained by Qatu and Leissa (1990). Theoretical value has been calculated using Equation 5.

$$F_{ij} = [\lambda_{ij} / (2\pi a2)] \times [E_{11}h^3/\rho]^{0.5} \qquad (5)$$

Where
$\lambda_{ij}$ = Non-Dimensional frequency
$\rho$ = Density
$E_{11}$ = Modulus of elasticity
h = Thickness
μ = Poisson ratio
a = Length

Figures 10 and 11 represents the variation of natural frequencies as a function of angle of twist for different ply orientations and corresponding analytical solution (Qatu and Leissa, 1990) are also presented in the graph. Figure 10 shows the results for composite laminate with an aspect ratios of b/h=20 and 100.

**Table 4. Frequencies for E-glass/epoxy (angle of twist Φ=0$^0$ and aspect ratio (b/h=20))**

| Ply angle (θ°) | Mode Number | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| | Ansys | Theory | Ansys | Theory | Ansys | Theory |
| 0 | 414.8 | 416.26 | 803.07 | 818.82 | 2146.1 | 2184.8 |
| 15 | 394.65 | 396.22 | 816.92 | 833.83 | 2169.8 | 2212.41 |
| 30 | 350.8 | 352.5 | 837.93 | 856.76 | 2115.3 | 2159.19 |
| 45 | 308.3 | 309.77 | 832.62 | 852.08 | 1888.9 | 1917.85 |
| 60 | 280.39 | 281.39 | 789.57 | 807.07 | 1727.4 | 1747.39 |
| 75 | 268.08 | 268.69 | 735.53 | 750.02 | 1660.6 | 1676.19 |
| 90 | 265.29 | 265.77 | 711.46 | 724.65 | 1646.46 | 1661.02 |

**Table 5. Frequencies for E-glass/epoxy (angle of twist Φ=15$^0$ and aspect ratio (b/h=20))**

| Ply angle (θ°) | Mode Number | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| | Ansys | Theory | Ansys | Theory | Ansys | Theory |
| 0 | 412.78 | 410.41 | 1033.6 | 1067.9 | 2216.2 | 2251.41 |
| 15 | 394.54 | 392.76 | 1039.3 | 1073.94 | 2231.7 | 2273.89 |
| 30 | 352.05 | 351.08 | 1035.8 | 1070.57 | 2077.7 | 2138.98 |
| 45 | 308.39 | 307.57 | 1003 | 1036.69 | 1832.2 | 1880.9 |
| 60 | 279.34 | 278.05 | 946.64 | 977.77 | 1667.9 | 1765.55 |
| 75 | 266.75 | 264.99 | 891.87 | 919.95 | 1599.1 | 1631.79 |
| 90 | 263.96 | 262.07 | 868.84 | 895.67 | 1583.9 | 1615.44 |

**Table 6. Frequencies for E-glass/epoxy (angle of twist $\Phi=30^0$ and aspect ratio (b/h=20))**

| Ply angle (θ°) | Mode Number | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| | Ansys | Theory | Ansys | Theory | Ansys | Theory |
| 0 | 407.56 | 391.05 | 1484.4 | 1661.84 | 2184.5 | 2300.5 |
| 15 | 390.53 | 375.68 | 1476 | 1651.43 | 2112.6 | 2230.79 |
| 30 | 349.47 | 337.46 | 1432.6 | 1596.78 | 1911.7 | 2021.9 |
| 45 | 305.43 | 295.2 | 1352.4 | 1503.9 | 1679 | 1771.06 |
| 60 | 275.8 | 265.6 | 1273 | 1417.98 | 1516.9 | 1590 |
| 75 | 263.26 | 252.6 | 1215.1 | 1357.39 | 1446.6 | 1510.5 |
| 90 | 260.59 | 249.75 | 1192.7 | 1333.65 | 1192.7 | 1491.6 |

**Table 7. Frequencies for E-glass/epoxy (angle of twist $\Phi=45^0$ and aspect ratio (b/h=20))**

| Ply angle (θ°) | Mode Number | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| | Ansys | Theory | Ansys | Theory | Ansys | Theory |
| 0 | 398.03 | 351.73 | 1881.7 | 2022.8 | 1895.9 | 2522.7 |
| 15 | 382.51 | 338.39 | 1792.4 | 1949.16 | 1914.6 | 2516.49 |
| 30 | 343.03 | 304.77 | 1630.7 | 1763.8 | 1842 | 2432.56 |
| 45 | 299.5 | 566.62 | 1452 | 1553.47 | 1701.7 | 2264.87 |
| 60 | 269.88 | 239.38 | 1323.6 | 1402.73 | 1581.9 | 2121.04 |
| 75 | 257.38 | 227.3 | 1264 | 1333.29 | 1513.7 | 2043.53 |
| 90 | 254.67 | 224.7 | 1249.2 | 1316.16 | 1490.1 | 2018.12 |

**Table 8. Frequencies for E-glass/epoxy (angle of twist $\Phi=0^0$ and aspect ratio (b/h=100))**

| Ply angle (θ°) | Mode Number | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| | Ansys | Theory | Ansys | Theory | Ansys | Theory |
| 0 | 83.22 | 83.25 | 163.28 | 163.77 | 435.51 | 436.96 |
| 15 | 79.2 | 79.24 | 166.22 | 166.77 | 441.28 | 442.48 |
| 30 | 70.42 | 70.51 | 170.79 | 171.35 | 430.75 | 431.83 |
| 45 | 61.87 | 61.96 | 169.72 | 170.42 | 383.11 | 383.8 |
| 60 | 56.23 | 56.28 | 160.8 | 161.41 | 349.06 | 349.48 |
| 75 | 53.71 | 53.73 | 149.5 | 150 | 334.98 | 335.23 |
| 90 | 53.13 | 53.15 | 144.48 | 145.43 | 332 | 332.2 |

**Table 9. Frequencies for E-glass/epoxy (angle of twist $\Phi=15^0$ and aspect ratio (b/h=100))**

| Ply angle (θ°) | Mode Number | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| | Ansys | Theory | Ansys | Theory | Ansys | Theory |
| 0 | 82.85 | 82.12 | 503.22 | 508.96 | 619.03 | 639.74 |
| 15 | 79.69 | 79.04 | 481.97 | 487.75 | 616.34 | 636.97 |
| 30 | 79.75 | 71.22 | 431.43 | 436.93 | 596.2 | 616.53 |
| 45 | 62.72 | 62.31 | 377.21 | 382.03 | 559.59 | 579.34 |
| 60 | 56.3 | 55.9 | 340.41 | 344.53 | 529.62 | 548.97 |
| 75 | 53.5 | 53.06 | 324.8 | 328.49 | 514.35 | 533.59 |
| 90 | 52.89 | 52.37 | 321.42 | 324.99 | 509.37 | 528.58 |

**Table 10. Frequencies for E-glass/epoxy (angle of twist Φ=30$^0$ and aspect ratio (b/h=100))**

| Ply angle (θ°) | Mode Number | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| | Ansys | Theory | Ansys | Theory | Ansys | Theory |
| 0 | 81.8 | 78.29 | 458.33 | 473.55 | 970.76 | 1076.09 |
| 15 | 78.73 | 75.44 | 440.62 | 455.67 | 962.85 | 1065.68 |
| 30 | 70.934 | 68.09 | 396.41 | 410.52 | 934.57 | 1032.66 |
| 45 | 62.01 | 59.59 | 346.77 | 359.27 | 890.53 | 983.38 |
| 60 | 55.63 | 53.39 | 311.75 | 322.67 | 864.46 | 932.96 |
| 75 | 52.83 | 50.62 | 296.39 | 306.39 | 864.48 | 902.87 |
| 90 | 52.25 | 50 | 293 | 302.73 | 865.88 | 895.38 |

**Table 11. Frequencies for E-glass/epoxy (angle of twist Φ=45$^0$ and aspect ratio (b/h=100))**

| Ply angle (θ°) | Mode Number | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| | Ansys | Theory | Ansys | Theory | Ansys | Theory |
| 0 | 79.9 | 70.47 | 401.98 | 419.05 | 1174.5 | 1339.67 |
| 15 | 77.09 | 67.93 | 387.11 | 403.75 | 1161.4 | 1290.14 |
| 30 | 69.58 | 61.38 | 349.05 | 364.52 | 1099.3 | 1165.47 |
| 45 | 60.84 | 53.74 | 305.63 | 319.19 | 965.36 | 1023.72 |
| 60 | 54.53 | 48.1 | 274.45 | 236.27 | 870.74 | 920.76 |
| 75 | 51.75 | 45.58 | 260.48 | 271.39 | 828.72 | 873.51 |
| 90 | 51.12 | 45.02 | 257.3 | 268.02 | 819.34 | 862.53 |

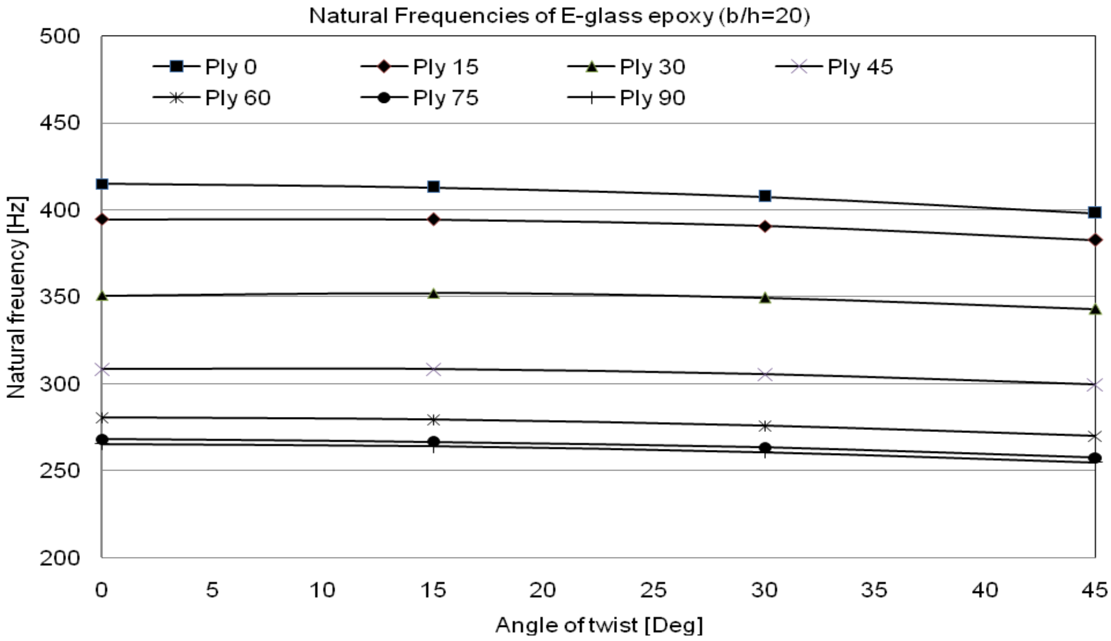


**Figure 10. Natural frequencies Vs Angle of twist for b/h=20**

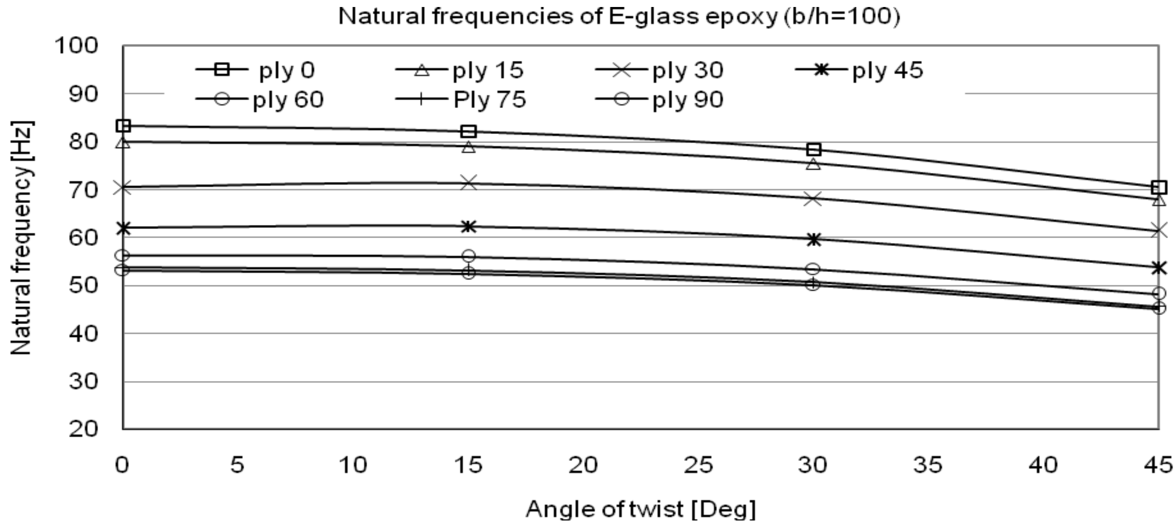


**Figure 11. Natural frequencies Vs Angle of twist for b/h=100**

## IV. GRAPHITE/EPOXY LAMINATED PLATE WITH THREE LAYERS AND PARAMETERS CONSIDERED

The laminated plate has a symmetric ply orientation [θ/-θ/θ] and is varied from 0° to 90° in steps of 15°. The angle of twist Φ is varied from 0° to 45° in steps of 15°. The two aspect ratios considered in this work are b/h=20 and 100. Figure 12 shows typical finite element meshed model. The laminated plate is assumed to be cantilever plate for which supporting boundary condition are shown in Figure 13.

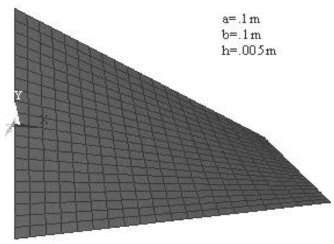


**Figure 12. FEM model of Graphite/epoxy**

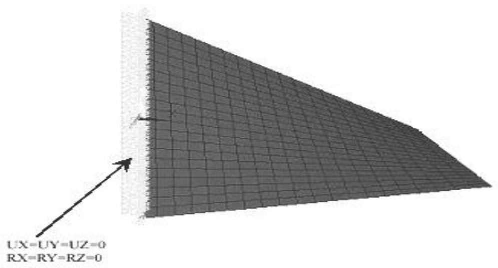


**Figure 13. Boundary condition of Graphite/epoxy**

The Material properties consider for the Graphite/epoxy laminated plates are shown in Table 12.

**Table 12. Graphite/Eepoxy Material properties**

| Material property | Value |
|---|---|
| Modulus of Elasticity | |
| $E_{XX}$ | $138 \times 10^9$ N/m$^2$ |
| $E_{YY}$ | $8.96 \times 10^9$ N/m$^2$ |
| $E_{ZZ}$ | $8.96 \times 10^9$ N/m$^2$ |
| Modulus of Rigidity | |
| $G_{XY}$ | $7.1 \times 10^9$ N/m$^2$ |
| $G_{YZ}$ | $7.1 \times 10^9$ N/m$^2$ |
| $G_{XZ}$ | $7.1 \times 10^9$ N/m$^2$ |
| Poisson ratio $\mu$ | 0.28 |
| Density $\rho$ | 1316 Kg/m$^3$ |
| Volume fraction | |
| Graphite | 55.60% |
| Epoxy | 44.40% |

### 4.1 Results and discussion

First three Eigen values and corresponding mode shapes were extracting for all the cases considered in this work. The results of the investigation are presented in Tables 13 to 20 for

various aspect ratios and angle of twists. The tables also incorporates the results of analytical investigations obtain by Qatu and Leissa (1990). Figures 14 and 15 shows the variations in natural frequencies as a function of angle of twist for different ply orientations.

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**Table 13. Frequencies for Graphite/epoxy (angle of twist $\Phi=0^0$ and aspect ratio (b/h=20))**

| Ply angle (θ°) | Mode Number | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| | Ansys | Theory | Ansys | Theory | Ansys | Theory |
| 0 | 819 | 829.16 | 1092.3 | 1118.36 | 2090.3 | 2150.75 |
| 15 | 700 | 710.34 | 1115.9 | 1146.48 | 2225.8 | 2302.16 |
| 30 | 520.72 | 530.33 | 1134.7 | 1171.9 | 2448.4 | 2546.71 |
| 45 | 369.56 | 375.75 | 1077.2 | 1119.25 | 2226.7 | 2324.08 |
| 60 | 267.87 | 270.79 | 928.52 | 962.15 | 1660.5 | 1692.38 |
| 75 | 221.27 | 221.89 | 764.04 | 782.46 | 1381.2 | 1391.82 |
| 90 | 210.88 | 211.06 | 690.08 | 702.6 | 1315.4 | 1322.25 |

**Table 14. Frequencies for Graphite/epoxy (angle of twist $\Phi=15^0$ and aspect ratio (b/h=20))**

| Ply angle (θ°) | Mode Number | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| | Ansys | Theory | Ansys | Theory | Ansys | Theory |
| 0 | 815.88 | 817.42 | 1374.2 | 1420.85 | 2256.9 | 2326.4 |
| 15 | 725.85 | 731.86 | 1411.4 | 1459.89 | 2419.4 | 2504.9 |
| 30 | 553.79 | 562.19 | 1399.7 | 1452.39 | 2597.7 | 2706.1 |
| 45 | 383.76 | 390.33 | 1273 | 1327.79 | 2189.5 | 2308.6 |
| 60 | 270.29 | 272.4 | 1071 | 1116.08 | 1612.8 | 1664.4 |
| 75 | 220.64 | 219.61 | 886.14 | 915.69 | 1332.3 | 1357.7 |
| 90 | 209.9 | 208.12 | 806.9 | 830.22 | 1265.4 | 1286.07 |

**Table 15. Frequencies for Graphite/epoxy (angle of twist $\Phi=30^0$ and aspect ratio (b/h=20))**

| Ply angle (θ°) | Mode Number | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| | Ansys | Theory | Ansys | Theory | Ansys | Theory |
| 0 | 805.8 | 778.47 | 1931.5 | 2135.84 | 2659 | 2852.38 |
| 15 | 739.77 | 713.77 | 1979.1 | 2187.18 | 2882.7 | 3105.81 |
| 30 | 568.68 | 564.15 | 1909.7 | 2114.8 | 2849.9 | 3094.08 |
| 45 | 391.63 | 393.68 | 1659.8 | 1838.4 | 2082.7 | 2276.6 |
| 60 | 269.84 | 267.53 | 1349.5 | 1473.98 | 1505.7 | 1631.01 |
| 75 | 218.14 | 210.41 | 1137.2 | 1233.99 | 1219.2 | 1303.26 |
| 90 | 207.31 | 198.35 | 1057.4 | 1173.37 | 1143.5 | 1187.95 |

**Table 16. Frequencies for Graphite/epoxy (angle of twist $\Phi=45^0$ and aspect ratio (b/h=20))**

| Ply angle (θ°) | Mode Number | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| | Ansys | Theory | Ansys | Theory | Ansys | Theory |
| 0 | 785.97 | 699.58 | 2411.4 | 3130.33 | 2971.3 | 3571.77 |
| 15 | 720.59 | 649.12 | 2412.4 | 3069.88 | 3243.6 | 3812.65 |
| 30 | 565.37 | 518.11 | 2240.4 | 2726.55 | 2786 | 3384 |
| 45 | 389.51 | 365.73 | 1778.2 | 2012.3 | 2127.4 | 2772 |
| 60 | 266.22 | 246 | 1291.4 | 1400.23 | 1679.6 | 2222.79 |
| 75 | 214.12 | 190.03 | 1055.7 | 1115.3 | 1461.5 | 1866 |
| 90 | 202.95 | 178.46 | 999.85 | 1648.69 | 1294.9 | 1733.4 |

**Table 17. Frequencies for Graphite/epoxy (angle of twist $\Phi=0^0$ and aspect ratio (b/h=100))**

| Ply angle (θ°) | Mode Number | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| | Ansys | Theory | Ansys | Theory | Ansys | Theory |
| 0 | 165.73 | 165.8 | 223.69 | 223.6 | 428.8 | 430.2 |
| 15 | 141.75 | 142 | 228.47 | 229.3 | 458.3 | 460.4 |
| 30 | 105.54 | 106.08 | 233.19 | 234.38 | 506.06 | 509.34 |
| 45 | 74.69 | 75.18 | 221.97 | 223.87 | 461.02 | 465.01 |
| 60 | 53.92 | 54.14 | 190.59 | 192.3 | 337.44 | 338.52 |
| 75 | 44.34 | 44.38 | 155.67 | 156.49 | 278.08 | 278.29 |
| 90 | 42.2 | 42.21 | 140.09 | 140.52 | 264.36 | 264.45 |

**Table 18. Frequencies for Graphite/epoxy (angle of twist $\Phi=15^0$ and aspect ratio (b/h=100))**

| Ply angle (θ°) | Mode Number | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| | Ansys | Theory | Ansys | Theory | Ansys | Theory |
| 0 | 164.69 | 163.5 | 757.37 | 781.69 | 925 | 955.3 |
| 15 | 152.5 | 151.5 | 761.6 | 785.12 | 922.3 | 940.5 |
| 30 | 121.94 | 121.66 | 678.08 | 692.6 | 781.61 | 808 |
| 45 | 85.9 | 86.15 | 482.52 | 506.76 | 670.73 | 695.62 |
| 60 | 57.65 | 57.77 | 336.49 | 341.95 | 552.97 | 574.4 |
| 75 | 44.66 | 44.38 | 270.22 | 273.63 | 471.14 | 489.49 |
| 90 | 42 | 41.64 | 255.5 | 258.3 | 440 | 457.09 |

**Table 19. Frequencies for Graphite/epoxy (angle of twist $\Phi=30^0$ and aspect ratio (b/h=100))**

| Ply angle (θ°) | Mode Number | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| | Ansys | Theory | Ansys | Theory | Ansys | Theory |
| 0 | 162.89 | 155.9 | 908.63 | 940.6 | 1225.6 | 1382.4 |
| 15 | 151.18 | 145.27 | 834.24 | 868.45 | 1252.5 | 1413.35 |
| 30 | 121.43 | 117.43 | 662.25 | 692.5 | 1190.9 | 1336.95 |
| 45 | 85.81 | 83.92 | 466.36 | 489.88 | 1049.8 | 1170.27 |
| 60 | 57.41 | 56.11 | 316.11 | 331.03 | 910.26 | 963.33 |
| 75 | 44.16 | 42.47 | 247.34 | 256.38 | 737.74 | 765.59 |
| 90 | 41.51 | 39.7 | 232.73 | 240.43 | 695.08 | 717.63 |

**Table 20. Frequencies for Graphite/epoxy (angle of twist $\Phi=45^0$ and aspect ratio (b/h=100))**

| Ply angle (θ°) | Mode Number | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| | Ansys | Theory | Ansys | Theory | Ansys | Theory |
| 0 | 159.03 | 140.29 | 795.57 | 831.39 | 1534.1 | 2080.26 |
| 15 | 148.39 | 131.11 | 737.55 | 774.98 | 1546.3 | 2110.58 |
| 30 | 119.72 | 106.73 | 590.7 | 625.03 | 1445.8 | 1963.16 |
| 45 | 84.85 | 76.57 | 419.12 | 444.86 | 1249.1 | 1401.65 |
| 60 | 56.81 | 51.09 | 283.22 | 299.62 | 892.14 | 964.19 |
| 75 | 43.47 | 38.33 | 218.5 | 227.9 | 696.66 | 737.46 |
| 90 | 40.67 | 35.73 | 204.54 | 212.82 | 652.69 | 687.64 |

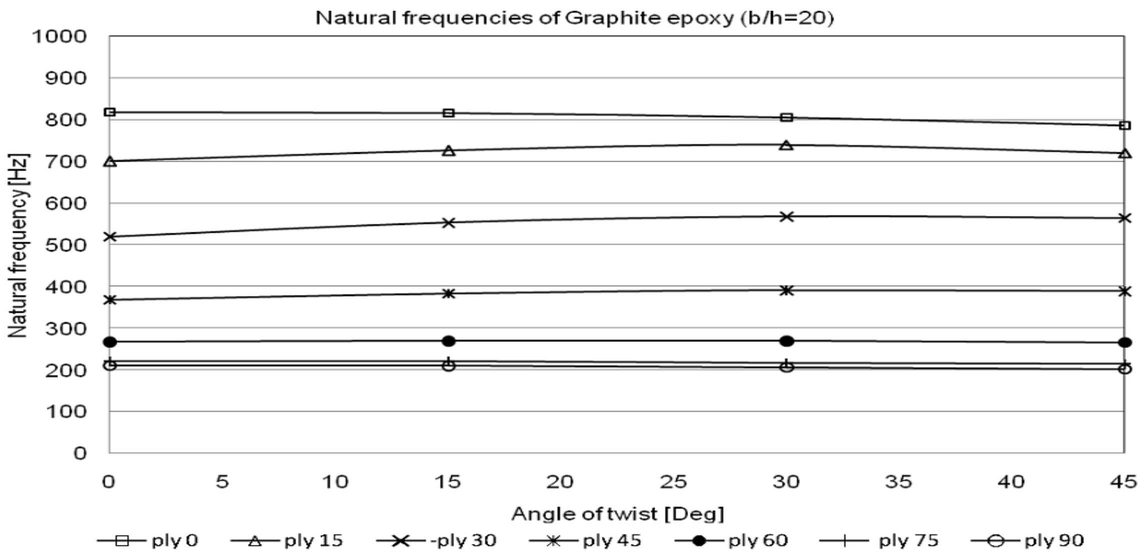


**Figure 14. Natural frequencies Vs Angle of twist for b/h=20**

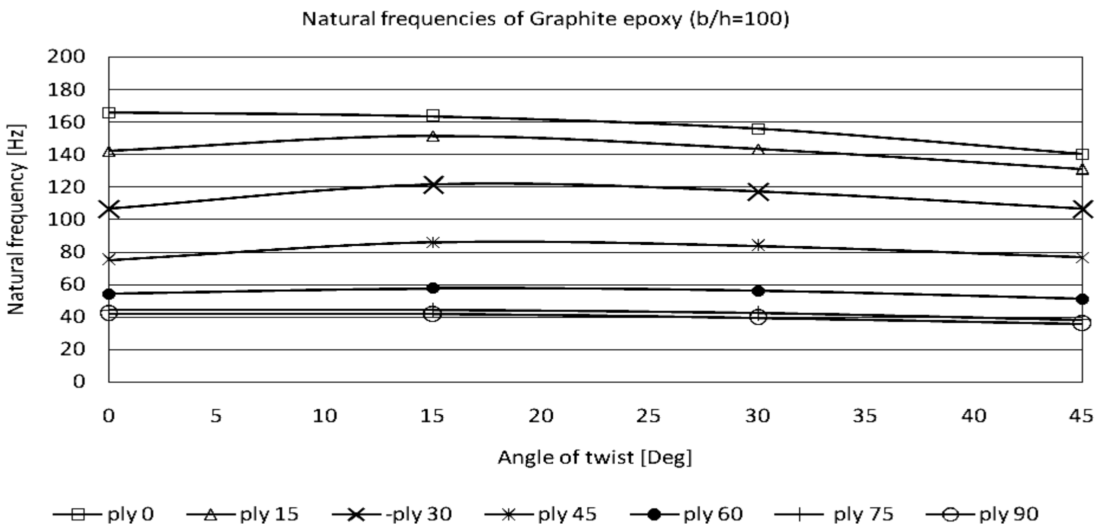


**Figure 15. Natural frequencies Vs Angle of twist for b/h=100**

## V. CONCLUSIONS

Following conclusions can be drawn from the results of the analysis:

- The Finite Element Method can be applied straight forwardly to investigate the practical problems such as studying the vibrational characteristic of laminated twisted cantilever plates. It provides reasonably accurate results with finite degrees of freedom when compared to some other methods.
- The peak values of fundamental frequencies of the twisted plates are observed when the fibers are perpendicular to the clamped edge in all cases studied.
- The fundamental natural frequency is found to be decreasing when the angle of twist value increases, which corresponding to the first bending mode.
- For E-glass/epoxy laminate with b/h=20 the natural frequency is constantly decreases when the ply orientation increases from 0° to 90°.
- For E-glass/epoxy laminate with b/h=100 and ply orientation 30° the natural frequency increases with increasing angle of twist from $0^{\circ}$ to $15^{\circ}$. After that it will decreases.
- For Graphite/epoxy laminate with aspect ratio b/h=20 and 100 having ply orientation ranging from 15° to 60° the natural frequency increases with increasing angle of twist.
- For Graphite/epoxy laminate with b/h=20 and 100 when the angle of twist increases, the natural frequencies gets reduced for laminates having ply orientations of 0° and 75º and 90°. But it increases for laminates having 15º and 60º ply orientations.
- FEM results are in close agreement with analytical solution [9]. The maximum deviation of 13.5% occurs at 45° angle of twist at $90^{0}$ ply orientation.

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## AUTHORS PROFILE


**Dr.V.Hariharan** is a Professor of Mechanical Engineering at Kongu Engineering College, Tamilnadu, India. He obtained his UG and PG degree from Madras University, Chennai and Ph.D from Anna University, Chennai .. His research Interest is Hybrid Natural fiber Composite materials, Condition Monitoring , Bearing Vibration analysis.

**Dr.G.Rajeshkumar** is an Associate Professor of Mechanical Engineering at PSG Institute of Technology and Applied Research, India, Tamilnadu India. He obtained his UG and PG Degree and Ph.D from Anna University, Chennai. His research Interest is Hybrid Natural fiber Composite materials.

**Dr.P.Thangavel** is an Associate Professor of Mechanical Engineering at Kongu Engineering ,Tamilnadu, India. He obtained his UG from Bharathiar University, PG Degree from NIT, Trichy, and Ph.D from Anna University, Chennai His research Interest is Refrigeration System, Renewable Energy Sources and Heat Transfer